1. Benchmark Model: Firms Pay the Adjustment Costs of Nominal Wages

A. Nash bargaining:
Firms and workers split the surplus of a match according to their bargaining powers. The asset value for an employed worker from a job is given by

\[ V_t^W = W_t h_t - \frac{P_t v(h_t)}{u_{c,t}} + \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( \frac{P_t}{P_{t+1}} \right) \left( 1 - \rho \right) V_{t+1}^W + \rho V_t^U \right], \]

(A1)

where the disutility from work is expressed in terms of the marginal utility of consumption (and hence it equals the marginal rate of substitution between consumption and labor). The asset value for an employed individual is the excess of current wage payments over the disutility from labor and the continuation value of staying employed or becoming unemployed next period. Those two events take place with probabilities \((1 - \rho)\) and \(\rho\), respectively.

Similarly, the asset value for an unemployed worker is given by

\[ V_t^U = P_t b + \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( \frac{P_t}{P_{t+1}} \right) \left( \theta_t q(\theta_t)(1 - \rho) V_{t+1}^W + (1 - \theta_t q(\theta_t)(1 - \rho)) V_{t+1}^U \right) \right], \]

(A2)

which equals unemployment benefits plus the continuation value. The latter is the weighted sum of the values of staying unemployed next period (which occurs with probability \(1 - \theta_t q(\theta_t)(1 - \rho)\)) and becoming employed (which occurs with probability \(\theta_t q(\theta_t)(1 - \rho)\)).

Finally, the value of a filled job for a firm (after suppressing the index \(j\)) is given by

\[ V_t^V = P_t m c z f(h_t) - W_t h_t - P_t \Phi_t^w + \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho) V_{t+1}^V \right], \]

(A3)

which suggests that the value of each match for the firm is equal to the flow value of its product net of wage payments and wage adjustment costs, plus the continuation value of that match in case of surviving exogenous separation.

The Nash bargaining problem is to choose \(W_t\) and \(h_t\) to maximize:

\[ (V_t^W - V_t^U)^\eta V_t^{1-\eta}, \]

(A4)
where \( \eta \) denotes the bargaining power of workers (and their share in the match surplus). In equilibrium, the value of posting a vacancy is zero and hence the threat point of firms is set to zero in the above formulation. The first-order condition with respect to \( W_t \) yields:

\[
(V_t^W - V_t^U) = \frac{\alpha_t}{(1 - \alpha_t)} V_t^V,
\]

(A5)

with \( \alpha_t = \frac{\eta}{\eta + (1 - \eta) \frac{\Delta_t^F}{\Delta_t^W}} \) being the effective bargaining power of workers, \( \Delta_t^W = \left( \frac{\partial V_t^W}{\partial W_t} - \frac{\partial V_t^U}{\partial W_t} \right) \) and

\[
\Delta_t^F = \frac{\partial V_t^V}{\partial W_t}.
\]

Combining the job creation condition (equation (15) in the text) with the asset value for the firm from a match (A3) gives

\[
V_t^V = P_t mc z_f(h_t) - W_t h_t - P_t \Phi_t^w + \frac{\gamma P_t}{q(\theta_t)}.
\]

(A6)

Also, substituting the expressions for \( V_t^V \), \( V_t^W \) and \( V_t^U \) in (A5) and using again the Job Creation condition yields the equation that characterizes the real wage setting:

\[
\frac{\omega_t}{1 - \omega_t} \left[ mc z_f(h_t) - w_t h_t - \Phi_t^w + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \frac{v(h_t)}{u_{c,t}} - b + E_t \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right],
\]

(A7)

which is equation (17) in the text.

Similarly, the first-order condition with respect to \( h_t \) yields:

\[
(V_t^W - V_t^U) = \frac{\Gamma_t}{(1 - \Gamma_t)} V_t^V,
\]

(A8)

where \( \Gamma_t = \frac{\eta}{\eta + (1 - \eta) \frac{\delta_t^F}{\delta_t^W}} \), \( \delta_t^W = -\frac{\partial V_t^W}{\partial h_t} \) and \( \delta_t^F = -\frac{\partial V_t^F}{\partial h_t} \).

Substituting the expressions for \( V_t^V \), \( V_t^W \) and \( V_t^U \) in (A5) and using again the Job Creation condition yields the equation that characterizes the determination of hours per worker:

\[
\frac{\Gamma_t}{1 - \Gamma_t} \left[ mc z_f(h_t) - w_t h_t - \Phi_t^w + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \frac{v(h_t)}{u_{c,t}} - b + E_t \left[ \frac{\Gamma_{t+1}}{1 - \Gamma_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right],
\]

(A9)

which is equation (18) in the text.
B. Efficient Allocations:
The problem of the social planner is to maximize:

\[ E_0 \sum_{i=0}^{\infty} \beta^t [u(c_t) - n_i v(h_t)], \]

subject to the sequence of the economy-wide resource constraints

\[ n_i z_{ij} f(h_i) - c_i - \nu_i = 0, \]

and the sequence laws of motion of employment

\[ (1 - \rho)[n_i + m(1 - n_i, v_i)] - n_{i+1} = 0 \]

Let, \( \lambda_{1,t} \) and \( \lambda_{2,t} \) denote the Lagrange multipliers on constraints (B2) and (B3), respectively. Then, the first-order condition with respect to \( c_i, v_i \) and \( n_{i+1} \), respectively, are

\[ u_{c,t} = \lambda_{1,t}, \]

\[ -\gamma \lambda_{1,t} + \lambda_{2,t} (1 - \rho) m_{v,t} = 0, \]

and,

\[ -\lambda_{2,t} = \beta v(h_{t+1}) + \beta E_t \lambda_{1,t+1} z_{t+1} f(h_{t+1}) + \beta (1 - \rho) E_t \lambda_{2,t+1} (1 - m_{u,t+1}) = 0. \]

Combining (B4)-(B6) gives:

\[ \frac{\gamma}{m_{v,t}} = \beta (1 - \rho) E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( z_{t+1} f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} + \frac{\gamma (1 - m_{u,t+1})}{m_{v,t+1}} \right) \right]. \]

Finally, after suppressing the expectation operator, condition (B7) can also be written as

\[ \frac{u_{c,t}}{\beta u_{c,t+1}} = (1 - \rho) \left[ \frac{z_{t+1} f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} + \frac{\gamma (1 - m_{u,t+1})}{m_{v,t+1}}}{\frac{\gamma}{m_{v,t}}} \right]. \]

The left hand side is the Intertemporal Marginal Rate of Substitution (IMRS), while the right hand side is Intertemporal Marginal Rate of Transformation (IMRT). Efficiency, thus, requires the IMRT being equal to the IMRS for all \( t \). Finally, this condition can also be written as

\[ 1 = (1 - \rho) E_t \left[ \frac{\beta u_{c,t+1}}{u_{c,t}} \left( z_{t+1} f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} + \frac{\gamma (1 - m_{u,t+1})}{m_{v,t+1}} \right) \right]. \]
C. The Intertemporal Wedge

In order to derive the intertemporal wedge, I make use of the vacancy-posting condition:

\[
\frac{\gamma}{q(\theta)} = \beta(1 - \rho)E_t \left\{ \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ mc_{t+1}z_{t+1}f(h_{t+1}) - w_{t+1}h_{t+1} - \Phi_{t+1}^w + \frac{\gamma}{q(\theta)} \right] \right\}, \tag{C1}
\]

and the real wage setting condition:

\[
\frac{\omega_t}{1 - \omega_t} \left[ mc_{t}z_{t}f(h_{t}) - w_{t}h_{t} - \Phi_{t}^w + \frac{\gamma}{q(\theta)} \right] = w_{t}h_{t} - \frac{v(h_{t})}{u_{c,t}} - b + E_t \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma}{q(\theta)} - \gamma \theta \right) \right]. \tag{C2}
\]

Using the properties of the matching function, we have \(q(\theta) = \frac{m_{v,t}}{1 - \zeta} \). Then, conditions (C1) and (C2) can, respectively, be written as

\[
\frac{\gamma(1 - \zeta)}{m_{v,t}} = \beta(1 - \rho)E_t \left\{ \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ mc_{t+1}z_{t+1}f(h_{t+1}) - w_{t+1}h_{t+1} - \Phi_{t+1}^w + \frac{\gamma(1 - \zeta)}{m_{v,t+1}} \right] \right\}, \tag{C3}
\]

and,

\[
\frac{\omega_t}{1 - \omega_t} \left[ mc_{t}z_{t}f(h_{t}) - w_{t}h_{t} - \Phi_{t}^w + \frac{\gamma(1 - \zeta)}{m_{v,t}} \right] = w_{t}h_{t} - \frac{v(h_{t})}{u_{c,t}} - b + E_t \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma(1 - \zeta)(\zeta - m_{u,t})}{\zeta m_{v,t}} \right) \right]. \tag{C4}
\]

Rearranging condition (C4) yields:

\[
w_{t}h_{t} = \omega_t \left[ mc_{t}z_{t}f(h_{t}) - \Phi_{t}^w + \frac{\gamma(1 - \zeta)}{m_{v,t}} \right] + (1 - \omega_t) \left[ \frac{v(h_{t})}{u_{c,t}} + b - E_t \left( \frac{\gamma(\zeta - m_{u,t})(1 - \zeta)\omega_{t+1}}{\zeta(1 - \omega_{t+1})m_{v,t}} \right) \right]. \tag{C5}
\]

After iterating one period ahead and collecting terms, equation (C5) can now be substituted into (C3) to yield:

\[
1 = (1 - \rho)E_t \left[ \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ mc_{t+1}z_{t+1}f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} - \Phi_{t+1}^w - b + \frac{\gamma(1 - \zeta)}{m_{v,t+1}} \left( 1 + E_{t+1}\left( \frac{\omega_{t+2}(\zeta - m_{u,t+1})}{\zeta(1 - \omega_{t+2})} \right) \right) \right] \right] \frac{\gamma}{m_{v,t} \left( 1 - \omega_{t+1} \right)}. \tag{C6}
\]

An analogous to condition (B8) can be obtained by rearranging terms in (C6) to get:

\[
\frac{u_{c,t+1}}{\beta u_{c,t+1}} = (1 - \rho) \left[ mc_{t+1}z_{t+1}f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} - \Phi_{t+1}^w - b + \frac{\gamma(1 - \zeta)}{m_{v,t+1}} \left( 1 + E_{t+1}\left( \frac{\omega_{t+2}(\zeta - m_{u,t+1})}{\zeta(1 - \omega_{t+2})} \right) \right) \right] \frac{\gamma}{m_{v,t} \left( 1 - \omega_{t+1} \right)}. \tag{C7}
\]

where the left-hand side is the intertemporal rate of substitution (IMRS) and the right-hand side is the intertemporal rate of transformation (IMRT).
2. Alternative Model: Firms Pay the Adjustment Costs of Nominal Wages

A short outline of the model when workers pay the adjustment costs of nominal wages is presented in this appendix.

A. Households:

The problem of households is to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - n_t v(h_t) + g(m_t) \right],$$

subject to the sequence of budget constraints of the form:

$$c_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + n_t \Phi_t^W = \frac{n_t h_t W_t}{P_t} + (1 - n_t) b + \frac{R_{t-1} B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} + \Theta_t,$$

where $n_t \Phi_t^W$ is the cost of adjusting nominal wages for all employed household’s members. Other variables and parameters are defined as in the main text. Hours per employed individual and wages are determined in a Nash bargaining process between firms and workers.

Household’s choices of $c_t$, $B_t$ and $M_t$ yield the following optimality conditions:

$$u_{c,t} = \beta R_t \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right),$$

and,

$$g_{m,t} = u_{c,t} \left( \frac{R_{t-1}}{R_t} \right),$$

which are the same optimality conditions as in the main text.

B. Firms:

Each firm chooses its price, vacancies and employment for next period to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} \left\{ \frac{P_{j,t}}{P_t} v_{j,t} - n_{j,t} w_{j,t} h_{j,t} - \nu_{j,t} - \frac{\varphi}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 \right\},$$

subject to the sequence of laws of motion of employment and the downward-sloping demand function for its product. Maximization yields the following Job Creation condition and Philips curve, respectively:

$$\frac{\gamma}{q(\theta_t)} = \beta (1 - \rho) E_t \left\{ u_{c,t+1} \right\},$$

$$1 - \varphi(\pi_t - 1) \pi_t + \beta \varphi \left[ \frac{u_{c,t+1}}{u_{c,t}} \right] (\pi_{t+1} - 1) \pi_{t+1} = \varepsilon (1 - m_c).$$
The new JC condition, therefore, differs from the original one by the absence of the cost of adjusting nominal wages.

C. Nash Bargaining:

The asset value for an employed worker from a job is given by

\[ V^W_t = W_t h_t - P_t v(h_t) - P_t \Phi^W_t + \beta E_t \left( \frac{u_{t+1}}{u_{t-1}} \right) \left( \frac{P_t}{P_{t+1}} \right) \left[ (1 - \rho) V^W_{t+1} + \rho V^U_{t+1} \right], \quad (C1) \]

where now the adjustment cost of nominal wage reduces the worker’s asset value from a match.

Similarly, the asset value for an unemployed worker is unchanged:

\[ V^U_t = P_t b + \beta E_t \left( \frac{u_{t+1}}{u_{t-1}} \right) \left( \frac{P_t}{P_{t+1}} \right) \left[ \theta_t q(\theta_t) (1 - \rho) V^W_{t+1} + (1 - \theta_t q(\theta_t) (1 - \rho) V^U_{t+1} \right], \quad (C2) \]

Finally, the value of a filled job for a firm is given by

\[ V^V_t = P_t m c_z f(h_t) - W_t h_t + \beta E_t \left( \frac{u_{t+1}}{u_{t-1}} \right) \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho) V^V_{t+1}, \quad (C3) \]

Bargaining over nominal wages gives the real wage setting equation:

\[ \frac{\omega_t}{1 - \omega_t} \left[ mc_z f(h_t) - w_t h_t + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \Phi^W_t - \frac{v(h_t)}{u_{t-1}} - b + E_t \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right], \quad (C4) \]

where \( \omega_t, \Delta_t^V \) and \( \Delta_t^W \) are defined as before. As in the case when firms pay the adjustment costs, DNWR drives a wedge between the effective and the ex-ante bargaining powers of workers.

Similarly, bargaining over hours per employed worker gives:

\[ \frac{\Gamma_t}{1 - \Gamma_t} \left[ mc_z f(h_t) - w_t h_t + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \Phi^W_t - \frac{v(h_t)}{u_{t-1}} - b + E_t \left[ \frac{\Gamma_{t+1}}{1 - \Gamma_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right], \quad (C5) \]

where \( \Gamma_t, \delta_t^V \) and \( \delta_t^W \) are defined as in the text.

Finally, conditions (13), (19)-(21) in the text remain unchanged. Therefore, they are not repeated here.

D. The Intertemporal Wedge

In order to derive the intertemporal wedge, I again use the job creation condition:

\[ \frac{\gamma}{q(\theta_t)} = \beta (1 - \rho) E_t \left( \frac{u_{t+1}}{u_{t-1}} \right) \left[ mc_{t+1} f(h_{t+1}) - w_{t+1} h_{t+1} + \frac{\gamma}{q(\theta_{t+1})} \right], \quad (D1) \]

and the real wage setting condition:
\[
\frac{\omega_i}{1-\omega_i} \left[ mc_z f(h_i) - w_i h_i + \gamma \frac{q(h_i)}{q(\theta)} \right] = w_i h_i - \Phi_i^w - v(h_i) - b + E_i \left[ \frac{\omega_{t+1}}{1-\omega_{t+1}} \left( \gamma \frac{q(\theta)}{q(\theta)} - \gamma \theta_i \right) \right],
\]  

(D2)

Conditions (D1) and (D2) can, respectively, be written as
\[
\gamma \left(1 - \zeta \right) \frac{m_{v,t}}{m_{v,t}} = \beta (1 - \rho) E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ mc_{t+1} z f(h_{t+1}) - w_{t+1} h_{t+1} + \gamma \left(1 - \zeta \right) \right],
\]

(D3)

and,
\[
\frac{\omega_i}{1-\omega_i} \left[ mc_z f(h_i) - w_i h_i + \gamma \left(1 - \zeta \right) \frac{m_{v,t}}{m_{v,t}} \right] = w_i h_i - \Phi_i^w - v(h_i) - b + E_i \left[ \frac{\omega_{t+1}}{1-\omega_{t+1}} \left( \gamma \left(1 - \zeta \right) (\zeta - m_{u,t}) \right) \right].
\]

(D4)

Rearranging condition (D4) yields:
\[
w_i h_i = \omega_i \left[ mc_z f(h_i) + \gamma \left(1 - \zeta \right) \frac{m_{v,t}}{m_{v,t}} \right] + (1 - \omega_i) \left[ v(h_i) - b + \Phi_i^w - E_i \left( \frac{\omega_{t+1}}{1-\omega_{t+1}} \left( \gamma \left(1 - \zeta \right) (\zeta - m_{u,t}) \right) \right) \right].
\]

(D5)

After iterating one period ahead and collecting terms, equation (D5) can now be substituted into (D3) to yield:
\[
1 = (1 - \rho) E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ \frac{mc_{t+1} z f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} - \Phi_{t+1}^w - \frac{\gamma \left(1 - \zeta \right)}{m_{v,t+1}} \left( 1 + E_{t+1} \left( \frac{\omega_{t+2} (\zeta - m_{u,t+1})}{\zeta (1 - \omega_{t+2})} \right) \right)}{\frac{\gamma}{m_{v,t}} \left( \frac{1 - \zeta}{1 - \omega_{t+1}} \right)} \right],
\]

(D6)

which, interestingly, is exactly the same as condition (25) in the text. Most importantly is that the adjustment cost of wages obtains the same sign as in the original model.

The corresponding equation that comes out of the social planner's problem is given by
\[
1 = (1 - \rho) E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ \frac{mc_{t+1} z f(h_{t+1}) - \frac{v(h_{t+1})}{u_{c,t+1}} + \frac{\gamma (1 - m_{u,t+1})}{m_{v,t+1}}}{\frac{\gamma}{m_{v,t}}} \right],
\]

(D7)

which is exactly equation (24) in the text.

The “intertemporal wedge” is implicitly defined by comparing the square brackets in condition (D6) with the square brackets in condition (D7). Clearly, the wedge in this version has the same form as in the original model in which firms pay all the costs of adjusting nominal wages (with, of course, \(\omega_i\) being potentially different).