Credit Constraints and the Government Spending Multiplier

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Abstract

This paper studies the government spending multiplier in a quantitative model with credit constraints. We have four key results. First, credit constraints generate a decline in the liquidity-trap government spending multiplier. This occurs partly because credit constraints weaken the “expected inflation channel” that has been central to large liquidity-trap spending multipliers in quantitative models. Second, this result holds even if the rise in government spending does not alter the tightness of the credit constraints. Third, the rise in government spending crowds out private borrowing, which leads to tighter credit constraints and even smaller spending multipliers. Fourth, with credit constraints, the liquidity-trap spending multiplier could be smaller than the spending multiplier during normal times.

Keywords: Government Spending Multiplier; Credit Constraints; Inflation; Liquidity Trap.

JEL Classification: E44; E62; H30; H50.

1. Introduction

This paper studies the effects of credit frictions on the government spending multiplier during periods of liquidity trap, as may occur when the nominal interest rate hits the zero lower bound (ZLB). The study extends an otherwise standard New Keynesian (NK) model with two types of households (impatient constrained borrowers and patient unconstrained savers), in which the credit friction arises because borrowing is constrained by collateral. We find that the liquidity-trap government spending multiplier with credit constraints is smaller than in a model that abstracts from such constraints.

To see the intuition behind this finding, consider first a case when the rise in government spending does not affect the tightness of the credit constraints (namely, the constraints were initially binding and remain binding to the same degree). Due to the credit constraints, the impatient agents consume less than they would in frictionless credit markets, which reflects a limited ability to borrow and obtain their desired level of consumption. Following a rise in government spending, their labor income rises, but the corresponding rise in consumption is smaller than it would be had the impatient households been able to borrow and smooth consumption freely. The limited rise in consumption of the impatient households reduces the response of total demand to a government spending shock. As a result, the increase in expected inflation is smaller compared to a model without credit constraints. When the nominal interest rate is fixed, the result is a smaller drop in...
the expected real interest rate. Consequently, consumption of both types of agents does not rise as much as it would in the absence of credit constraints, implying a smaller spending multiplier. This “neutral” case illustrates that the negative effect of credit constraints on the spending multiplier does not require credit constraints to become tighter following a rise in government spending.

We then show that the rise in government spending could make the credit constraints tighter, which further reduces demand and makes the spending multiplier smaller than in the “neutral” case. Under this scenario, inflation may decline, leading to a higher real interest rate. As a result, consumption of both types of agents would fall; the spending multiplier would be less than one in liquidity traps. Moreover, we illustrate that, for credit constraints to raise the spending multiplier, the constraints should become looser by certain magnitudes. Therefore, not any decline in the tightness of the credit constraint is sufficient to make the spending multiplier larger. An important implication of this result is that the credit constraints may become looser but the spending multiplier would still be smaller than in the model without these constraints.

The tighter credit constraints reflect the crowding out of private borrowing as well as the rise in taxes that are needed to finance government spending. Private borrowing, in turn, is reduced for a few reasons. First, the rise in government borrowing comes at the expense of private borrowing. Second, the reduced consumption by the constrained households induces a rise in their labor supply (a wealth effect), which in itself reduces their real wage. Labor supply also rises as the constrained agents attempt to make their credit constraints less tight. The weaker demand for products reduces labor demand and real wages of all households. The labor income of the constrained agents is thus reduced (relative to the model with no credit constraints), which allows for less borrowing. Third, to smooth consumption, the constrained households have to either borrow more and/or reduce their asset holdings. The value of collateral then declines at the same time that these agents would like to borrow more. Fourth, since taxes of the unconstrained households rise, the amount of loanable funds that they provide to the private sector is reduced. These factors combined lead to a decline in the borrowing of the constrained households, tighter credit constraints and, eventually, a smaller spending multiplier.

Our second main result is that the spending multiplier during liquidity traps is larger than the multiplier during normal times when credit constraints are absent. However, with credit constraints, the liquidity-trap multiplier may become smaller than the normal-times multiplier. This finding challenges the perception that government spending is more effective in periods of slack.

The finding regarding the behavior of inflation is significant: since the seminal work of Christiano et al. (2011), the quantitative literature on the spending multiplier has evolved around the behavior of inflation and expected inflation. This channel is often called the “expected inflation channel” of government spending whereby a rise in government spending generates a rise in expected inflation, which in turn reduces the expected real interest rate and boosts consumption. Christiano et al. (2011) show that the spending multiplier is large if the nominal interest rate does not respond to a rise in government spending. We show that this channel is markedly weakened or even fully eliminated by credit frictions.

This paper contributes to the recent quantitative literature on the spending multiplier. In a related study to ours, Carrillo and Poilly (2013) find that financial frictions increase the government spending multiplier, particularly at the ZLB. When the ZLB constraint binds, a rise in government spending reduces the real interest rate and allows for cheaper credit. Entrepreneurs use this time
to accumulate more capital, which in turn increases their collateral and reduces future costs. By doing so, a rise in government spending encourages investment and leads to a larger spending multiplier. In our study, credit constraints lead to a weaker demand, a smaller rise (or even a decline) in the inflation rate, and a smaller fall (or a rise) in the real interest rate. Consequently, government spending is less effective than it would be in the absence of credit constraints.

Using a New-Keynesian framework, Carlstrom et al. (2014) compare the spending multiplier when the monetary-fiscal expansion lasts a certain number of periods ("deterministic duration") with the multiplier when the expansion is stochastic. They find the size of the stochastic multiplier to be larger than the deterministic multiplier. Zubairy (2014) studies the government spending multiplier in a stochastic general equilibrium model that features distortionary taxes and finds that the impact government spending multiplier is slightly above one. Bouakez et al. (2017) find a multiplier of 2.3 in a model with public investment. Woodford (2011) shows that a spending multiplier of well above one is possible under extreme circumstance as such in the Great Depression era. However, under less extreme circumstance, the spending multiplier may not be much greater than one, and it could be less than one or even negative if the persistence of the fiscal stimulus after the financial disturbance ends is sufficiently large. Leeper et al. (2017) show that the averages of short-run output multipliers are similar across regimes, but they are considerably larger after 10 years under the passive money/active fiscal regime than the active money/passive fiscal regime.

Dupor and Li (2015) find that large spending multipliers require large responses of expected inflation to government spending, which does not align with their empirical evidence using U.S. data. Therefore, the expected inflation channel has no support in U.S. data or it has been too small to generate large fiscal multipliers. Our quantitative work suggests that credit constraints on households could be one explanation for the weak(er) "expected inflation channel". Ramey and Zubairy (2018) construct U.S. data for 1889-2013 to test whether government spending multipliers differ according to the amount of slack in the economy or being near the ZLB. They find that the amount of slack in the economy does not affect the size of the multiplier. Furthermore, their results for ZLB are mixed: the full sample indicates no larger spending multipliers near the zero lower bound, but a sub-sample that excludes the rationing periods of WWII indicates, in some cases, larger spending multipliers in the zero lower bound. Using a panel of OECD countries, Boehm (forthcoming) finds that, while the point estimates of the government spending multipliers differ between ZLB and non-ZLB episodes, the standard errors were large and, thus, equality between the multipliers at and away of the ZLB cannot be rejected. The model-based results that we obtain in this paper are largely consistent with these findings, which could suggest that the rise in the tightness of credit conditions in downturns reduces the effectiveness of fiscal policy.

Using Japanese data, Miyamoto et al. (2018), Miyamoto et al. (2018) show that, at the ZLB, the government spending multiplier is considerably larger and the expected inflation rate rises by more than in non-ZLB episodes. Wieland (2019) finds that, in contrast to the predictions of the New Keynesian model, negative supply shocks are not expansionary at the ZLB (the analyses are based on the 2011 Great East Japan Earthquake and oil supply shocks). He then demonstrates that modifications of the model that are consistent with the empirical evidence would also overturn other unusual policy predictions at the ZLB, such as large fiscal multipliers.

More generally, our paper adds to the literature that questions the effectiveness of economic policies in the presence of credit frictions. Using a general equilibrium model, in which agents face
uninsurable idiosyncratic income risk and borrowing constraints, McKay et al. (2016) show that, at the ZLB, forward guidance becomes considerably less powerful in stimulating the economy in the incomplete-market setup than in the standard macro model without borrowing constraints. Alpanda et al. (2019) find that the impact of monetary policy shocks on output and most other macroeconomic and financial variables is smaller during periods of economic downturns, high household debt and high interest rates. It is then shown that a small-scale theoretical model that highlights the presence of collateral and debt-service constraints on household borrowing and refinancing could potentially rationalize these facts.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy. Section 3 presents analytical analyses and Section 4 presents numerical results. Robustness analyses are presented in Section 5. Section 6 concludes.

2. The Model

The economy is populated by a continuum of infinitely-lived patient (unconstrained) households, impatient (constrained) households, intermediate-good firms, final-good firms, a monetary authority and a government. The impatient households are of measure \( \omega \in (0, 1) \) and the patient households are of measure \((1 - \omega)\). The impatient agents borrow from financial intermediaries and face borrowing constraints with real estate serving as collateral. Real estate is introduced in this model so that borrowers have an asset to back their borrowing. In this respect, the use of real estate as collateral is common in the literature; see Kiyotaki and Moore (1997), Iacoviello (2005) and Gerali et al. (2010), among others.

2.1. Patient (Unconstrained) Households

The representative patient household derives utility from consumption \( c_{P,t} \), supplies labor \( n_{P,t} \) and has utility from real estate \( h_{P,t} \) (e.g. in the form of housing) in each period \( t \). The patient households also make nominal deposits \( D_t \) at the financial intermediary with a gross nominal interest rate of \( R_{D,t} \), and lend \( B_{P,t} \) to the government at a nominal gross interest rate of \( R_t \). These households also own the firms and the financial intermediaries. The problem of the representative patient household is given by:

\[
\max_{\{B_{P,t}, D_t, c_{P,t}, h_{P,t}, n_{P,t}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{P,t}^{1-\sigma} - \chi_{P} n_{P,t}^{1+\nu} + \psi_{P} \ln h_{P,t}}{1 - \sigma} \right)
\]

with \( \beta(< 1) \) being the subjective discount factor of the patient households, \( \mathbb{E}_t \) is the expectation operator, \( \sigma \) is the consumption curvature parameter and \( \nu \) is the inverse of the labor supply elasticity. The scaling parameters \( \chi_{P} \) and \( \psi_{P} \) measure the relative weights of labor and real estate in the utility function, respectively.

Maximization is subject to the sequence of budget constraints:

\[
c_{P,t} + \frac{Q_t h_{P,t}}{P_t} + \frac{B_{P,t}}{P_t} + \frac{D_t}{P_t} + T_{P,t} = w_{P,t} n_{P,t} + \frac{R_{t-1} B_{P,t-1}}{P_t} + \frac{R_{D,t-1} D_{t-1}}{P_t} + \frac{Q_t h_{P,t-1}}{P_t} + \Pi_{P,t}
\]

with \( T_{P,t} \) being the real taxes that are paid by these agents, \( w_{P,t} \) is the real wage, \( \Pi_{P,t} \) are the real profits of the representative patient household from owning the intermediaries (banks) and firms, \( Q_t \) is the nominal price of real estate and \( P_t \) is the aggregate price level.
Optimization with respect to \( B_{P,t}, D_{t}, c_{P,t}, h_{P,t} \) and \( n_{P,t} \) gives the following conditions:

\[
c_{P,t}^{\sigma} = \beta R_{t} \mathbb{E}_{t} \left( \frac{c_{P,t+1}^{\sigma}}{\pi_{t+1}} \right) 
\]

(3)

\[
c_{P,t}^{\sigma} = \beta R_{D,t} \mathbb{E}_{t} \left( \frac{c_{P,t+1}^{\sigma}}{\pi_{t+1}} \right) 
\]

(4)

\[
\chi_{P} n_{P,t}^{\nu} c_{P,t}^{\sigma} = w_{P,t} 
\]

(5)

\[
q_{t} c_{P,t}^{\sigma} = \psi_{P} h_{P,t} + \beta \mathbb{E}_{t} (q_{t+1} c_{P,t+1}^{\sigma}) 
\]

(6)

with \( \pi_{t} = \frac{P_{t}}{P_{t-1}} \) being the gross price inflation rate and \( q_{t} = \frac{Q_{t}}{P_{t}} \) denoting the real price of real estate. Condition (3) is the consumption Euler equation, condition (4) governs the choice of deposits, (5) is the labor supply condition, and equation (6) is an asset-pricing condition that governs the demand for real estate. Notice that conditions (3) and (4) imply:

\[
R_{D,t} = R_{t} 
\]

(7)

which, as in Romer and Romer (1990) and Carrillo and Poilly (2013) among others, reflects the fact that deposits and government bonds are perfect substitutes from the perspective of these households.

2.2. Impatient (Constrained) Households

Each period, the representative impatient household derives utility from consumption \( c_{I,t} \) and from real estate \( h_{I,t} \), supplies labor \( n_{I,t} \), and borrows \( B_{I,t} \) from the financial intermediary at a gross nominal interest rate of \( R_{I,t} \). Borrowing, however, is subject to a borrowing constraint as in the standard limited enforcement problem of Kiyotaki and Moore (1997). In particular, borrowing is tied to the nominal value of real estate of these agents. The objective of the representative impatient household is then to:

\[
\max_{\{B_{I,t}, c_{I,t}, h_{I,t}, n_{I,t}\}_{t=0}^{\infty}} \mathbb{E}_{0} \sum_{t=0}^{\infty} \delta^{t} \left( c_{I,t}^{1-\sigma} \frac{n_{I,t}^{1+\nu}}{1+\nu} + \psi_{I} \ln h_{I,t} \right) 
\]

(8)

with \( \delta (< \beta) \) being the subjective discount factor of the impatient households, and \( \chi_{I} \) and \( \psi_{I} \) are scaling parameters. Maximization is subject to the sequence of budget constraints:

\[
c_{I,t} + \frac{Q_{I} h_{I,t}}{P_{t}} + \frac{R_{I,t-1} B_{I,t-1}}{P_{t}} + T_{I,t} = w_{I,t} n_{I,t} + \frac{B_{I,t}}{P_{t}} + \frac{Q_{I} h_{I,t-1}}{P_{t}} 
\]

(9)

and the following credit (borrowing) constraint:

\[
B_{I,t} \leq \tau Q_{I} h_{I,t-1} 
\]

(10)

where \( w_{I,t} \) is the real wage of these households, \( T_{I,t} \) denotes real taxes and \( \tau \) being the loan-to-value ratio. Letting \( \lambda_{I,t} \) be the Lagrange multiplier on the credit constraint (10), we obtain the following Euler equation, labor supply condition and the demand for real estate condition, respectively:
\[ c^\sigma_{I,t} = \delta R_{I,t} E_t \left( \frac{c^\sigma_{I,t+1}}{\pi_{t+1}} \right) + \lambda_{I,t} \]  
(11)

\[ \chi_{I,t} c^\sigma_{I,t} = w_{I,t} \]  
(12)

\[ q_t c^\sigma_{I,t} = \frac{\psi_{I,t}}{h_{I,t}} + \delta E_t \left[ q_{t+1} c^\sigma_{I,t+1} + \tau q_{t+1} \lambda_{I,t+1} \right] \]  
(13)

Condition (11) shows how the tightness of the credit constraint affects the variations in consumption of the constrained households. Throughout the paper, we assume that the credit constraint is always binding (hence \( \lambda_{I,t} > 0 \) for all \( t \)).

By dividing both sides of condition (10) by the price level \( (P_t) \), the credit constraint in real terms reads:

\[ b_{I,t} \leq \tau q_t h_{I,t-1} \]  
(14)

with \( b_{I,t} = \frac{B_{I,t}}{P_t} \) being the real value of borrowing by the representative impatient household.

### 2.3. Final-Good Firms

Firms in this sector operate in a perfectly competitive environment. They purchase a continuum of intermediate goods from intermediate-good producers, indexed by \( j \in (0, 1) \), and assemble them into final goods using the following technology:

\[ y_{t} = \left( \int_0^1 y_{j,t} \frac{\varepsilon}{\varepsilon - 1} dj \right)^{\frac{1}{\varepsilon - 1}} \]  
(15)

with \( y_{j,t} \) being the quantity of intermediate-good \( j \) that is purchased by a final-good firm and \( \varepsilon > 1 \) is the elasticity of substitution between differentiated types of intermediate goods. Profit maximization gives the following downward-sloping demand function for the product variety \( j \):

\[ y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\varepsilon} y_t \]  
(16)

where \( P_t = \left( \int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{1/\varepsilon} \) is the Dixit-Stiglitz aggregate price level.

### 2.4. Intermediate-Good Firms

Firms in this sector are monopolistically competitive, owned by the unconstrained households and of a unit mass. Each firm \( j \) hires labor \( N_{j,t} \) from both types of households to produce a differentiated product using the following technology:

\[ y_{j,t} = N_{j,t} \]  
(17)

Following Gerali et al. (2010), we let \( N_{j,t} = N_{I,j,t}^{\alpha} N_{P,j,t}^{1-\alpha} \), with \( \alpha < 1 \) being the share of labor supplied by the impatient households and \( 1-\alpha \) the share of the patient households. Each period, a firm \( j \) has the probability \( 1-\eta \) of changing its price as in Calvo (1983). With this characterization, the formal problem of the intermediate-good firm \( j \) is to:

\[ \max_{\{N_{I,j,t}, N_{P,j,t}, P_t\}} \mathbb{E}_0 \sum_{s=0}^{\infty} (\beta \eta)^s c^\sigma_{P_t+s} \left( \frac{P_{P_t}}{P_t} \right)^{\alpha} N_{I,j,t+s}^{\alpha} N_{P,j,t+s}^{1-\alpha} - w_{I,t+s} N_{I,j,t+s} - w_{P,t+s} N_{P,j,t+s} \]  
(18)
subject to (16)-(17) and the definition of \( N_{j,t} \). The labor demand conditions for both types of agents are, respectively, given by:

\[
(1 - \alpha)mc_{j,t}N_{j,t}^{\alpha}N_{P,j,t}^{-\alpha} = w_{P,t} \tag{19}
\]

\[
\alpha mc_{j,t}N_{j,t}^{\alpha-1}N_{P,j,t}^{-\alpha} = w_{I,t} \tag{20}
\]

with \( mc_{j,t} \) being the real marginal cost. For each labor input, the real marginal cost equals the real wage over the marginal product of the respective input.

As is standard in the NK literature, the first-order condition that yields the optimal price chosen by the firm \( (P_{j,t}^*) \) is given by:

\[
P_{j,t}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\Psi_{1,t}}{\Psi_{2,t}} \quad \text{where } \Psi_{1,t} \text{ and } \Psi_{2,t} \text{ take the following recursive forms:}
\]

\[
\Psi_{1,t} = c_{P,t}^{-\sigma}m_{c}y_{t} + \beta \eta \mathbb{E}_{t}(\Psi_{1,t+1}^{\varepsilon} + \varepsilon \Psi_{2,t}^{\varepsilon-1}) \tag{23}
\]

\[
\Psi_{2,t} = c_{P,t}^{-\sigma}y_{t} + \beta \eta \mathbb{E}_{t}(\Psi_{2,t+1}^{\varepsilon-1}) \tag{24}
\]

Substituting for \( P_{j,t}^* \) using condition (22) and using the definition of the aggregate price level, \( P_{t}^{1-\varepsilon} = (1 - \eta)(P_{t}^*)^{1-\varepsilon} + \eta P_{t-1}^{1-\varepsilon} \), then gives:

\[
\eta \pi_{t}^{\varepsilon-1} = 1 - (1 - \eta)\left(\frac{\varepsilon}{\varepsilon - 1} \frac{\Psi_{1,t}}{\Psi_{2,t}}\right)^{1-\varepsilon} \tag{25}
\]

As usual with Calvo pricing, the fact that not all firms can adjust their prices each period leads to a price dispersion, which is given by:

\[
\Delta_{t} = \int_{0}^{1} \left( \frac{P_{j,t}}{P_{t}} \right)^{-\varepsilon} dj \tag{26}
\]

Defining \( \pi_{t}^{*} = \frac{P_{t}^{*}}{P_{t-1}} \), condition (22) can be written in terms of inflation rates as follows:

\[
\frac{\pi_{t}^{*}}{\pi_{t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\Psi_{1,t}}{\Psi_{2,t}} \tag{27}
\]

Then, the price dispersion evolves according to the following equation:

\[
\Delta_{t} = (1 - \eta)\left( \frac{\pi_{t}^{*}}{\pi_{t}} \right)^{-\varepsilon} + \eta \pi_{t}^{\varepsilon} \Delta_{t-1} \tag{28}
\]

As expected, \( \Delta_{t} = 1 \) when all firms update prices (i.e. \( \eta = 0 \), in which case \( P_{t}^{*} = P_{t} \)).
2.5. The Financial Intermediary

The financial intermediation sector is perfectly competitive, and thus we may think of a representative intermediary. For simplicity, we also abstract from stickiness in deposits or loans. The intermediary receives deposits from the patient households (savers) and lends to the impatient households (borrowers). Letting $b_{I,t} = B_{I,t}/P_t$ be the real value of borrowing by each impatient household and $d_t = D_t/P_t$ be the real deposits of each patient household, the total real amount of deposits is $m_t = (1-\omega)d_t$ and the total real amount of loans is $l_t = \omega b_{I,t}$. Then, the problem of the intermediary is to:

$$\max_{(l_t,m_t)} \sum_{t=0}^{\infty} \beta^t c_{P,t}^{-\sigma} \left( R_{I,t} l_t - R_{D,t} m_t \right)$$

subject to the intermediary’s balance sheet $l_t = m_t$, where we assume that the required reserve ratio is zero. The solution to the intermediary’s problem yields:

$$R_{I,t} = R_t$$

which is as expected with no monopolistic power or adjustment costs in this sector.\(^1\)

2.6. Fiscal Policy

Government spending is financed via taxes and bonds:

$$g_t + \frac{R_{t-1}b_{G,t-1}}{\pi_t} = \omega T_{I,t} + (1-\omega)T_{P,t} + b_{G,t}$$

with $b_{G,t}$ being the real value of the total supply of bonds by the government.

Let $\bar{x}$ be the non-stochastic (i.e. deterministic) steady state of any variable $x_t$. Then, government spending evolves according to the following AR(1) process:

$$\ln \left( \frac{g_t}{\bar{g}} \right) = \rho \ln \left( \frac{g_{t-1}}{\bar{g}} \right) + u_t$$

with $\rho$ denoting the persistence parameter of government spending and $u_t \sim N(0,\sigma_u^2)$.

Taxes follow the following rules:

$$\ln \left( \frac{T_{P,t}}{T_P} \right) = \rho_B \ln \left( \frac{b_{G,t-1}}{b_G} \right) + \rho_G \ln \left( \frac{g_t}{\bar{g}} \right)$$

$$\ln \left( \frac{T_{I,t}}{T_I} \right) = \rho_B \ln \left( \frac{b_{G,t-1}}{b_G} \right) + \rho_G \ln \left( \frac{g_t}{\bar{g}} \right)$$

with $\rho_B$ and $\rho_G$ being the coefficients of bonds and government spending, respectively. Two comments on the assumption of lump-sum taxation are in order. First, it aligns with previous studies on the government spending multiplier. Second, if the model were to allow for distortionary taxes, then changes in tax rates that co-occur with changes in government spending will make it harder

\(^1\)In a previous version of this paper, we showed that allowing for market power and adjustment costs in the intermediation sector (and therefore deviations between $R_{I,t}$ and $R_t$) does not change the key results of the paper. These results are available upon request.
to disentangle the contribution of government spending to variations in output (tax rates might rise to cover the rise in government spending or might be lowered as another means of stimulating the economy). For these reasons, we abstract from distortionary taxation.

2.7. Monetary Policy
Monetary policy is governed by the following interest-rate rule:

\[ \ln \left( \frac{R_t}{R} \right) = \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y} \right) \] (35)

with \( \phi_\pi \) and \( \phi_y \) being the coefficients of inflation and output, respectively.

2.8. Market Clearing
The economy-wide resource constraint is given by:

\[ y_t = \omega c_{I,t} + (1 - \omega) c_{P,t} + g_t \] (36)

The real estate market clears:

\[ \omega h_{I,t} + (1 - \omega) h_{P,t} = 1 \] (37)

where, as in Gerali et al. (2010) and Iacoviello (2005), real estate is in a fixed supply. One justification of this assumption is that real estate can be given the interpretation of land.

The bonds market clears (\( b_{P,t} = b_{P,t} \)):

\[ (1 - \omega) b_{P,t} = b_{G,t} \] (38)

The loanable funds market clears:

\[ (1 - \omega) d_t = \omega b_{I,t} \] (39)

The total amounts of labor that are demanded from both types of households by all firms are \( \int_0^1 N_{I,j,t} dj = N_{I,t} \) and \( \int_0^1 N_{P,j,t} dj = N_{P,t} \). Then, the clearing conditions in the labor market are:

\[ N_{I,t} = \omega n_{I,t} \] (40)

\[ N_{P,t} = (1 - \omega) n_{P,t} \] (41)

Aggregate output is then given by:

\[ y_t = \Delta_t^{-1} N_{I,t}^{\alpha} N_{P,t}^{1-\alpha} \] (42)

which makes clear the role of price dispersion in determining aggregate output, as is standard in the New-Keynesian model. By combining conditions (38)-(39), the share of total private credit in total loanable funds is given by:

\[ b^{Share}_{I,t} = \frac{\omega b_{I,t}}{1 - \omega} \frac{1}{b_{P,t} + d_t} \] (43)

Finally, due to perfect competition in the intermediation sector, the profits are zero in equilibrium. In the intermediate-good sector, total profits are given by \( \int_0^1 \Pi_{j,t} dj = \Pi_t \). This is also
the total amount of profits that all patient households have. Therefore, the share of each patient household in total profits satisfies:

$$\Pi_t = (1 - \omega)\Pi_{P,t}. \quad (44)$$

This concludes the description of the model.

2.9. The Private-Sector Equilibrium

**Definition 1** (Equilibrium). Given the exogenous process of $g_t$ (32), the private-sector equilibrium is a sequence of allocations $\{b_{I,t}, b_{G,t}, b_{P,t}, c_{I,t}, c_{P,t}, d_t, h_{I,t}, h_{P,t}, mc_t, n_{I,t}, n_{P,t}, N_{I,t}, N_{P,t}, q_t, R_{D,t}, R_{I,t}, R_t, T_{I,t}, T_{P,t}, w_{I,t}, w_{P,t}, y_t, \lambda_{I,t}, \pi_t, \pi^*_t, \Delta_t, \Psi_{1,t}, \Psi_{2,t}\}$ that satisfy the equilibrium conditions (3)-(6),(9)-(11)-(14),(19)-(20),(23)-(25),(27)-(28),(30)-(31),(33)-(42).

Our definition of the private-sector equilibrium includes the budget constraint of the impatient households, the budget constraint of the government and the resource constraint. The budget constraint of the patient households then follows by the Walras’ Law.

2.10. The Log-Linearized Model

This subsection outlines the log-linearized version of the model, which we later use to derive the spending multiplier analytically. First, for analytical analysis, it is useful to re-write $\lambda_{I,t} = c_{I,t}^{-\sigma}\mu_t$. In turn, since $c_{I,t}^{-\sigma}$ is positive and finite, the credit constraint will bind if $\mu_t > 0$. Second, by letting $\overline{x}_t$ be the log deviation of any variable $x_t$ from its non-stochastic steady state ($\overline{x}$), we can write every variable as: $x_t = \overline{x}_t(1 + \overline{x}_t)$. Third, in the following system of equations, we impose the equilibrium conditions $\overline{R}_{I,t} = \overline{R}_{D,t} = \overline{R}_t$ and thus use only $\overline{R}_t$. We also use the fact that $\overline{N}_{I,t} = \overline{n}_{I,t}$ and $\overline{N}_{P,t} = \overline{n}_{P,t}$ to reduce the number of conditions. Then, we obtain the log-linearized system of equations (B.1)-(B.21) in Appendix B.

Without credit constraints, conditions (B.1)-(B.2), (B.4)-(B.5), (B.9)-(B.13), (B.17) and (B.21) constitute a system of 11 equations with 11 variables ($c_{I,t}, c_{P,t}, n_{I,t}, n_{P,t}, w_{I,t}, w_{P,t}, y_t, mc_t, R_t, g_t, \pi_t$). By solving this system, we can pin down the spending multiplier. This system, however, does not include taxes and bonds; therefore, without credit frictions, one can derive the spending multiplier independently of the method via which government spending is financed. Alternatively, without credit-constrained agents, the economy is essentially populated by one type of agents. It is straightforward to show that the spending multiplier can then be derived regardless of the assumption about the financing of government spending.

Also, combining the deterministic steady-state versions of conditions (3) and (11) yields $\overline{\lambda}_t = \overline{c}_{I,t}^{-\sigma}(\frac{\beta - \delta}{\beta})$. Since $\beta > \delta$ and $\overline{c}_{I,t}$ is positive and finite, the credit constraint binds at the steady state of the model. Furthermore, $\overline{\mu}$ appears in the log-linearized model and, therefore, it is important for deriving the spending multiplier analytically. The combination of the steady-state versions of conditions (3) and (11) gives $\overline{\mu} = \frac{\beta - \delta}{\beta} > 0$. Therefore, even though $\overline{\mu}$ is not a parameter, it is a function of two structural parameters, and it will be constant as long as neither $\beta$ nor $\delta$ is changed.

3. The Government Spending Multiplier- Analytical Analysis

This section presents analytical analysis regarding the government spending multiplier (the derivations are available in the online appendix). In order to obtain tractable expressions, we make the following simplifying assumptions: the weight of impatient agents in labor equals their weight in
the population \((\alpha = \omega)\) and the monetary authority responds to inflation only \((\phi_y = 0)\). Following the literature (e.g. Christiano et al., 2011 and Carlstrom et al., 2014), the model is solved using the Method of Undetermined Coefficients. Specifically, define the change in output following a change in government spending as:

\[
\hat{y}_t = A_y \hat{g}_t \tag{45}
\]

Then, the government spending multiplier is given by \(A_y/g\). The spending multiplier depends on the response of total consumption \((c_t)\) to changes in government spending: \(\hat{c}_t = A_c \hat{g}_t\). From the resource constraint, the response of output to spending shocks may also be re-written as \(A_y = g + (1 - g)A_c\). The response of consumption to spending shocks determines whether the multiplier is larger than, less than or equals one. If \(A_c = 0\), then the multiplier is one \((A_y = g)\). Similarly, when all output is allocated to government spending \((g = 1)\), the spending multiplier is one.

Define also the response of \(\mu_t\) to government spending shocks as:

\[
\hat{\mu}_t = Z \hat{g}_t \tag{46}
\]

The coefficient \(Z\) will be crucial for the analysis, and we outline its derivation from first principles in the online appendix for normal times and liquidity traps. One could then substitute the value of \(Z\) to obtain a spending multiplier as a function of structural parameters only, but it leads to intractable expressions that make the exposition of ideas harder. Therefore, in this section we keep \(Z\) as part of the spending multipliers’ expressions.

3.1. The Government Spending Multiplier During Normal Times

In normal times, the nominal interest rate is allowed to adjust following shocks to government spending. Under the standard assumption of \(\phi_\pi > 1\), the Method of Undetermined Coefficients yields the following solution:

\[
\frac{A^{NT}_y}{g} = 1 + \frac{-\nu \kappa (\phi_\pi - \rho)(1 - g) - \omega M \left(\frac{\beta - \delta}{\delta (1 - \rho)}\right) Z^{NT}}{\sigma(1 - \beta \rho)(1 - \rho) + \kappa (\phi_\pi - \rho) \left[\sigma + \nu (1 - g)\right]} \tag{47}
\]

with \(NT\) denoting “normal times”, \(\kappa = \frac{(1 - \eta)(1 - \beta \eta)}{\eta}\) is the slope of the log-linearized Philips curve and \(M = (1 - \beta \rho)(1 - \rho) + \kappa (\phi_\pi - \rho)\) is the spending multiplier is a function of parameters and the response of \(\mu_t\) to government spending shocks \((Z^{NT})\). The first term in the numerator of condition \((47)\) is standard in New-Keynesian models, while the second term in the numerator reflects the effects of credit constraints. As we show later, \(M > 0\) in our baseline analysis and we proceed under this assumption. Similarly, the response of inflation to a government spending shock is given by:

\[
A^{NT}_\pi = \frac{\kappa \sigma \nu (1 - \rho) g - \omega \kappa (\sigma + \nu c_t) \left(\frac{\beta - \delta}{\sigma}\right) Z^{NT}}{\sigma(1 - \beta \rho)(1 - \rho) + \kappa (\phi_\pi - \rho) \left[\sigma + \nu (1 - g)\right]} \tag{48}
\]

The first term in the numerator and the entire denominator of expression \((47)\) are not affected by the credit constraint. We, thus, can safely conduct the comparisons between the models with and without credit constraints by examining the effects of the credit constraint on the numerator of this expression. If \(Z^{NT} > 0\), then the numerator with credit constraints is lower than in a model that abstracts from credit frictions; therefore, the spending multiplier with credit frictions
is smaller. The opposite result holds when $Z^{NT} < 0$. Clearly, if $Z^{NT} = 0$, then the normal-times spending multiplier will be independent of credit constraints.

Regarding the behavior of inflation following government spending shocks, the credit constraints appear only in the numerator of expression (48). With $Z^{NT} > 0$, the numerator with credit frictions is lower than in a model that abstracts from credit frictions. Therefore, with credit frictions, the response of inflation to spending shocks is smaller. The opposite holds with $Z^{NT} < 0$, and the response of inflation to spending shocks is not affected by credit constraints if $Z^{NT} = 0$. Therefore, one way through which credit constraints could affect the government spending multiplier is through their effects on the response of the inflation rate to government spending shocks.

The behavior of total consumption following government spending shocks helps in understanding the potential effects of credit constraints on the spending multiplier. The response of consumption to a government spending shock is given by:

$$A^{NT} = -\frac{\phi\pi - \rho}{\sigma(1 - \rho)} A^{NT} - \frac{\omega(\beta - \delta)}{\sigma(1 - \rho)\delta} Z^{NT} \tag{49}$$

The second term on the right-hand side of condition (49) represents a direct effect of credit frictions on the consumption multiplier. To the extent that $Z^{NT} > 0$, this effect is negative. Credit frictions also have an indirect effect, which is captured by the first term on the right-hand side, and it operates through the response of inflation to spending shocks; inflation is affected by credit constraints and, by extension, so is consumption. If $Z^{NT} > 0$, then inflation is negatively affected by credit constraints, and the indirect effect on consumption is positive.

With $Z^{NT} > 0$, credit constraints reduce (or limit the rise of) consumption of the impatient agents, which reduces the effectiveness of fiscal policy (the “direct effect”): due to the credit constraints, these agents may not be able to raise their consumption (or can only raise it to a certain degree) following a rise in government spending. On the other hand, the decline (or muted rise in consumer spending) limits the rise in inflation and inflation expectations that are originally induced by the rise in government spending. Since the nominal interest rate rises by more than one-for-one with expected inflation, the rise in the expected real interest rate is muted. This in itself limits the negative impact on consumption, hence inducing a larger spending multiplier compared to the model without credit constraints (this is the “indirect effect” of credit constraints). Since $\phi_{\pi} > \rho$, these two effects contradict. When $Z^{NT} < 0$, the opposite occurs: the direct effect raises the response of consumption to a spending shock while the indirect effect reduces it. Therefore, the net effect on the spending multiplier depends on the relative strength of each channel.

### 3.2. The Government Spending Multiplier During Liquidity Trap

We now present the liquidity-trap government spending multiplier, which occurs when the nominal interest rate is fixed ($\bar{R}_t = 0$). For this experiment, we may think of the nominal interest rate as hitting the zero-lower bound and staying there for a certain period of time. As is standard in the literature, this may occur because of a discount factor shock that pushes the nominal interest rate to the ZLB and follows a Markov process. With probability $\varrho$, the nominal interest rate remains at the ZLB next period, and with probability $1 - \varrho$ the shock expires and the system reverts back to normal. The system of equations that we use has no endogenous state variables, and thus all endogenous control variables jump on impact, but they revert back to their steady-state values.
once the shock expires. For this reason, we can write $E_t \hat{\pi}_{t+1} = \varrho \hat{\pi}_t$ and $E_t \hat{c}_{t+1} = \varrho \hat{c}_t$.\(^2\)

The solution then yields the following liquidity-trap spending multiplier:

$$\frac{A_{LT}^y}{g} = 1 + \frac{\nu \kappa (1 - g) - \omega \left[ (1 - \beta \varrho)(1 - g) - \varrho \kappa \right] c_t + (1 - g) \varrho \kappa}{\sigma (1 - \beta \varrho)(1 - g) - \varrho \kappa} Z_{LT}^T$$

and the response of inflation to a government spending shock:

$$A_{LT}^\pi = \frac{\kappa \sigma \nu (1 - g) g - \omega \kappa (\sigma + \nu c_t)}{\sigma (1 - \beta \varrho)(1 - g) - \varrho \kappa} Z_{LT}^T$$

where $LT$ denotes “liquidity trap”. Following Christiano et al. (2011) and subsequent studies, we let $\varrho = \rho$ so that the probability that the nominal interest will not change next period equals the persistence parameter of government spending, which enables better comparisons between the liquidity-trap multiplier and the normal-times multiplier.

With a fixed nominal interest rate, the denominator of condition (50) might be negative. As has been discussed in Christiano et al. (2011) and Carlstrom et al. (2014), when the denominator is negative, we enter the “indeterminacy” region and multiple equilibria are possible. To ensure the existence of a unique stationary equilibrium, we assume that the denominator is positive. On this basis, the parameter $\kappa$ is set so that $\kappa < \frac{\sigma (1-\beta \varrho)(1-g)}{\varrho \sigma + \nu (1-g)}$.\(^3\) Also, when the nominal interest rate is fixed, we have $\hat{\pi}_t = A_{LT}^\pi \hat{g}_t$, and the expected inflation rate is given by $E_t \hat{\pi}_{t+1} = \varrho A_{LT}^\pi \hat{g}_t$, implying that the response of the expected inflation rate to government spending shocks is proportional to the response of the actual inflation rate to government spending shocks. The same applies to the real interest rate $\hat{r}_t$ and the expected real interest rate $E_t \hat{r}_{t+1}$.

According to condition (50), if $Z_{LT}^T > 0$, then the government spending multiplier with credit constraints will be smaller than in the absence of these constraints. If $Z_{LT}^T < 0$, the opposite result holds, and if $Z_{LT}^T = 0$, then credit constraints do not affect the size of the government spending multiplier. These arguments also hold for the response of inflation to a government spending shock.

The response of consumption to a government spending shock is given by:

$$A_{LT}^c = \frac{\theta}{\sigma (1 - \varrho)} A_{LT}^\pi - \frac{\omega (\beta - \delta)}{\sigma (1 - \varrho)} Z_{LT}^T$$

The second term on the right-hand side of condition (52) represents a direct effect of credit frictions on consumption. To the extent that $Z_{LT}^T > 0$, this effect is negative: credit constraints reduce the ability of the constrained households to borrow and expand their consumption. The indirect effect is represented by the first right-hand side term of this condition, and it operates through the response of inflation to spending shocks. Since credit frictions reduce the response of inflation to spending shocks (condition (51)), $A_{LT}^\pi$ with credit frictions is smaller than in a model with no frictions. When the nominal interest rate is fixed, the corresponding decline in the expected

\(^2\)With probability $\varrho$, and for any variable $x_t$, we have $\hat{x}_t \neq 0$. With probability $(1 - \varrho)$, the shock expires and the system reverts back to the steady state: $\hat{x}_t = 0$. Therefore, the expected value is given by $E_t \hat{x}_{t+1} = \varrho \hat{x}_t + (1 - \varrho) \cdot 0 = \varrho \hat{x}_t$.

\(^3\)The determinacy condition is not affected by the share of the impatient households ($\omega$) due to the assumption that both types of households have the same consumption curvature parameter ($\sigma$). Without this assumption, the denominator of the spending multiplier becomes a function of $\omega$.\(^4\)
real interest rate will be smaller, which reduces the impact on consumer spending. If credit frictions are sufficiently large, inflation might decline, which would increase the real interest rate and further depress consumption. In either case, the indirect effect on consumption is negative too. Therefore, both effects push towards a lower spending multiplier. On the other hand, with \( Z_{LT}^{LT} < 0 \), the direct and indirect effects push for a larger liquidity-trap multiplier. This is an important finding: while in normal times the effect of credit frictions on consumption depends on the relative strength of each channel, during liquidity traps both channels operate in the same direction.

While the overall effect of credit constraints on consumption is negative when \( Z_{LT}^{LT} > 0 \), it is still possible that consumption would rise following a rise in government spending (this may happen when \( Z_{LT}^{LT} \) is not sufficiently large). Therefore, since the Ricardian Equivalence does not hold in the presence of credit constraints, fiscal policy is effective in raising consumption: in this case, as a result of the rise in government spending, labor income rises and so does consumption. This channel, however, is weakened by the existence of credit constraints.

We briefly comment on some of the parameters that affect the spending multiplier. The share of the constrained households (\( \omega \)) negatively affects the spending multiplier, and with \( \omega = 0 \), credit frictions are eliminated from the expression of the spending multiplier. The spending multiplier is also influenced by the gap between the discount rates of both types of households (\( \beta - \delta \)) as it affects the tightness of the credit constraint; other things equal, the larger this gap, the greater the credit friction, and the smaller is the spending multiplier. The steady-state government-output ratio (\( g \)) affects the spending multiplier; with \( g = 1 \), we have \( c_i = 0 \), and the spending multiplier is one. Price rigidity (\( \kappa \)) matters greatly for the size of the spending multiplier. With fully flexible prices (\( \kappa \to \infty \)), the liquidity trap multiplier is independent of credit frictions and less than one. In the other extreme, with fully rigid prices (\( \kappa = 0 \)), the spending multiplier without credit constraints during liquidity traps is one (and the expected inflation channel is nullified), but different than one with credit constraints. Therefore, with credit constraints, even though the expected inflation channel is shut down (i.e. no indirect effect of credit constraint on the spending multiplier), the direct effect of credit constraints is still operative. The spending multiplier is also affected by the persistence of government spending (\( \rho \)); in particular, the spending multiplier is a decreasing function of \( \rho \). As noted by Christiano et al. (2011), the intuition for this result is that the present value of taxes that are associated with a given rise in government spending is increasing in \( \rho \). Therefore, the negative wealth effect on consumption is increasing in \( \rho \) as well.

Finally, let \( \tilde{Z} \) be the response of \( \mu_t \) to government spending shocks such that the response of inflation to government spending shocks is zero in liquidity traps and normal times (\( A_{\pi}^{LT} = A_{\pi}^{NT} = 0 \)). Then, under the assumption \( \rho = \varrho \), we obtain:

\[
\tilde{Z} = \frac{\sigma \nu (1 - \rho) g \delta}{\omega (\beta - \delta)(\sigma + \nu c_i)}
\]  

which is positive and the same for both liquidity traps and normal times. Conditions (47) and (50) then give:

\[
\frac{A_{y}^{LT}}{g} = \frac{A_{y}^{NT}}{g} = \frac{\sigma}{\sigma + \nu c_i}
\]  

The spending multipliers are thus equal and less than one. However, since we have not yet determined the sign of \( Z_{LT}^{LT} \) (outside this particular case), at this stage of the paper we cannot
determine whether the liquidity-trap spending multiplier is smaller or larger when credit constraints are present. We, thus, shall wait for the quantitative analysis to make that determination.

3.3. A Thought Experiment: $\lambda_{I,t} = 0$

This subsection considers a thought experiment where the Lagrange multiplier on the credit constraint does not respond to government spending shocks ($\lambda_{I,t} = 0$). As shown in Appendix C, the spending multiplier and the response of inflation to a spending shock during liquidity traps are, respectively, given by:

$$A_{LT}^y = \frac{(\beta - \rho \delta)[\sigma(1 - \beta \varrho)(1 - \varrho) - \varrho \kappa \sigma] + \omega \varrho \kappa(\beta - \delta)\sigma}{g(\beta - \rho \delta)[\sigma(1 - \beta \varrho)(1 - \varrho) - \varrho \kappa[\sigma + \nu(1 - g)]]} + \omega \varrho \kappa(\beta - \delta)(\sigma + \nu c_i)$$  \hspace{1cm} (55)

$$A_{LT}^\pi = \frac{\kappa \nu(1 - \varrho)(\beta - \rho \delta)\sigma g}{(\beta - \rho \delta)[\sigma(1 - \beta \varrho)(1 - \varrho) - \varrho \kappa[\sigma + \nu(1 - g)]]} + \omega \varrho \kappa(\beta - \delta)(\sigma + \nu c_i)$$  \hspace{1cm} (56)

The first term in the numerator and the first term in the denominator of condition (55) are positive by virtue of the determinacy condition. As $\omega$ rises, the numerator rises by less than the denominator; therefore, the spending multiplier is lower for higher values of $\omega$. The spending multiplier with credit-constrained households ($\omega > 0$) is thus smaller than the spending multiplier with no credit-constrained households ($\omega = 0$). Similarly, when $\omega$ rises, the response of inflation to government spending shocks is weaker.

The liquidity-trap spending multiplier is smaller when the economy is populated by constrained agents even if the rise in government spending does not tighten the credit constraints. This finding does not hang on any of our simplifying assumptions (other than $\alpha = \omega$) or the method of financing the rise in government spending. It reflects the weaker response of inflation to spending shocks: a weaker rise by inflation implies a smaller fall in the real interest rate, which induces a smaller stimulative effect of government spending on consumption of both types of agents; see conditions (C.2)-(C.3).

We close this subsection with three comments. First, the response of consumption of the constrained households to spending shocks is smaller than the response of the consumption of the unconstrained households. As we show in the appendix, the relative response of the constrained households’ consumption is given by:

$$\frac{A_I}{A_P} = \frac{\delta - \rho \delta}{\beta - \rho \delta}$$  \hspace{1cm} (57)

Since $\delta < \beta$, we have $A_I < A_P$. And, since $A_\pi$ is declining in $\omega$, $A_I$ and $A_P$ are also declining in $\omega$. Therefore, the response of total consumption to spending shocks ($A_c$) is declining in $\omega$. As a result, the spending multiplier is smaller.

Second, using the Phillips curve, the response of the real marginal cost to a government spending shock is given by $A_{mc} = (1 - \beta \varrho)A_\pi / \kappa$. Since $A_\pi$ is smaller with credit constraints, $A_{mc}$ is also smaller (as expected). But, since $\bar{w}_t = \bar{m} \bar{c}_t$, the response of the real wage is also weaker. In addition, using the labor supply conditions and the production function, we can write: $A_{mc} = \nu g + [\sigma + \nu(1 - g)] A_c$. Therefore, a weaker response by the marginal cost implies a weaker response by total consumption ($c_t$). If $mc_t$ declines following a government spending shock, then $c_t$ must decline as well.
Third, the response of the Lagrange multiplier on the credit constraint to spending shocks is given by
\[ \lambda_{t} = \mu_{t} - \sigma c_{t}. \] If \( \lambda_{t} = 0 \), then \( \mu_{t} = \sigma c_{t} \); the size of \( Z^{LT} \) is not necessarily zero. Under this scenario, the effects of the credit constraint remain in place even if the credit constraint does not become tighter following spending shocks; the credit constraint was binding and remains binding to the same degree (which is its steady-state level). Furthermore, a positive \( \mu_{t} \) implies a positive \( c_{t} \). This is an interesting case: \( Z^{LT} \) is positive (which means a smaller spending multiplier than in the model without credit constraints) even though consumption of the impatient agents rises. In a way, \( Z^{LT} \) serves as an “index” regarding the effects of credit constraints on the spending multiplier. If it is positive, then the spending multiplier is smaller even if the credit constraint does not become tighter. In fact, \( Z^{LT} \) may be positive even after the credit constraint becomes looser.

3.4. Can the Liquidity-Trap Spending Multiplier Be Less Than One?

In this subsection, we discuss special cases that enable us to make concrete statements about the sizes of the spending multiplier in the presence of credit-constrained households. In particular, we discuss conditions under which the liquidity-trap spending multiplier is less than one. The full details can be found in the online appendix. In what follows, we summarize our results in the form of propositions.

**Proposition 1.** Suppose that the parameter values are such that there is determinacy. Suppose also that the nominal interest rate is fixed at its steady-state value (\( \bar{R}_{t} = 0 \)) and that the economy is populated with a measure \( \omega \) of credit-constrained households. If prices are fully rigid (\( \kappa = 0 \)) or government spending is not persistent (\( \rho = 0 \)), then the liquidity-trap spending multiplier is less than one if and only if
\[ (1 + \nu)(c_{i} + g) > 1. \]

*Proof:* With \( \kappa = 0 \) or \( \rho = 0 \), the government spending multiplier is given by:
\[ \frac{A^{LT}_{y}}{g} = \frac{\omega \sigma (c_{i} + g) - c_{i} + \omega c_{i}}{\omega \sigma (c_{i} + g) - c_{i} + \omega c_{i} (1 + \nu)(c_{i} + g)} \] (58)

Therefore, if prices are fully rigid, the spending multiplier would be less than one if and only if \( (1 + \nu)(c_{i} + g) > 1. \) QED

In the absence of credit-constrained households (\( \omega = 0 \)), the spending multiplier is exactly one, in line with previous studies. However, the introduction of credit constraints could affect this result, and the size of the spending multiplier depends on \( (1 + \nu)(c_{i} + g) \). At this stage, however, we cannot determine the size of \( (1 + \nu)(c_{i} + g) \), and thus we relegate the determination of the size of this expression to the numerical analysis.

**Proposition 2.** Suppose that the parameter values are such that there is determinacy. Suppose also that the nominal interest rate is fixed at its steady-state value (\( \bar{R}_{t} = 0 \)). Consider a model with a measure \( \omega \) of hand-to-mouth households. If prices approach full flexibility (\( \kappa \to \infty \)), then the government spending multiplier during liquidity traps is less than one for any value of \( \omega \).

*Proof:* Letting \( \kappa \to \infty \) we obtain:
\[ \frac{A^{LT}_{y}}{g} = \frac{\sigma}{\sigma + \nu (1 - g)} \] (59)
Therefore, the liquidity-trap spending multiplier is less than one. QED.

The case of fully flexible prices constitutes one example for which it is possible to show the size of liquidity-trap multiplier analytically. It also illustrates that credit frictions do not raise the spending multiplier in liquidity traps.

**PROPOSITION 3.** Suppose that the parameter values are such that there is determinacy. Suppose also that the nominal interest rate is fixed at its steady-state value ($\bar{R}_t = 0$). Consider a model with a measure $\omega$ of hand-to-mouth consumers. If $\omega = 1$, then the government spending multiplier during liquidity traps is less than one.

**Proof:** With $\omega = 1$ we obtain:

$$\frac{A_{LT}^{yg}}{g} = \frac{(\sigma - 1)}{\sigma - 1 + (1 - g)(1 + \nu)}$$

and, therefore, the spending multiplier is less than one. QED.

A comment on the case of $\omega = 1$ is in order: when all households do not save, the government does not have a source for borrowing. Therefore, we view this result as illustrating what happens when the share of HtM households is very large (e.g. approaches unity).

4. Numerical Evaluation

4.1. Parameterization

Table 1 reports the parameter values that we use in the benchmark analyses. Some of these parameters values (i.e. $\beta, \sigma, \nu, \kappa, \rho, g, \phi, \phi_y, \varepsilon$) are widely accepted in the literature on the government spending multiplier. Therefore, we elaborate on other parameters. The discount factor of the impatient households is set based on the values in the literature with credit frictions: it ranges from 0.95 (as in Carlstrom and Fuerst, 1997) to 0.98 (Iacoviello, 2005 and Monacelli, 2009). We, thus, set it to the middle of this range. We set $\omega = 1/3$ so that the share of the impatient households matches the share of hand-to-mouth consumers in Kaplan and Violante (2014). The parameter $\alpha$ is set to equal the share of the impatient households in the population ($\alpha = \omega$). The loan-to-value ratio ($\tau$) is standard in the literature.

The disutility-of-labor parameters $\chi_P$ and $\chi_I$ are set so that the deterministic steady-state of total labor supply by each type of households is 0.21 (which corresponds to a workweek of 35 hours, the average number of weekly hours worked in the U.S. over the period 1964:Q1-2016:Q4). Therefore, $\bar{N}_P = \bar{N}_I = 0.21$, and $\bar{y} = 0.21$. The scaling parameters $\psi_P$ and $\psi_I$ are set so that the steady-state value of real estate for each type of agents is 1.

The deterministic steady-state value of government spending ($\bar{g}$) is such that it constitutes 20% of deterministic steady-state GDP, which corresponds to the average government spending-GDP ratio over the period 1967:Q1-2016:Q4. The steady-state value of debt $b_t$ is obtained so that $\bar{b}/\bar{y}$ is 55.4%, which is the average of the federal debt as percentage of GDP for the same period. To

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4 As we show in the online appendix, the spending multipliers that are presented in Propositions 2-3 apply also to normal times. Therefore, under the conditions that are discussed in these propositions, government spending is not more effective during liquidity traps than normal times.
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Table 1: Values of the parameters - benchmark analyses.

obtain the coefficients of the tax rules, we first multiply condition (33) by $1 - \omega$ and (34) by $\omega$. Then, combining the resultant two conditions, defining $T_t = \omega T_{I,t} + (1 - \omega) T_{P,t}$ and assuming that both types of households are paying the same amount of taxes give:

$$
\ln\left(\frac{T_t}{T}\right) = \rho_B \ln\left(\frac{b_{G,t-1}}{b_G}\right) + \rho_G \ln\left(\frac{g_t}{g}\right)
$$

To estimate this equation, we use quarterly data for total Federal tax revenues, Federal debt and Federal government spending over the period 1967:Q1-2016:Q4. We conduct estimation using both the Generalized Method of Moments (GMM) and Ordinary Least Squares (OLS), and the results are highly similar (the full estimation results can be found in Table A.1 in the Appendix). We thus use the GMM estimates in our benchmark calibration.

The parameter values imply that the maximum value of $\kappa$ to satisfy the determinacy condition is 0.043. Our benchmark value of $\kappa$ is thus consistent with this requirement. The implied price rigidity parameter is $\eta = 0.83$, which is in the upper bound of the typically used values of this parameter in New Keynesian models. As noted above, we use this value of $\kappa$ to be consistent with previous studies on the government spending multiplier even though it is implies a relatively high degree of price rigidity.

4.2. First-Pass Numerical Results

The goal of this subsection is to provide some numerical support of the analytical analysis and to build some intuition for the results that we obtain later in the paper. We primarily focus on how the introduction of credit frictions affect the size of the spending multiplier and the response of inflation to spending shocks. To obtain the results of this subsection, we use condition (47) for normal times and condition (50) for liquidity traps. Since the value of $Z$ is endogenous (in particular, it depends on $\omega$), we consider the size of the credit friction as a whole, which we define as $\omega \left(\frac{\beta - \delta}{\delta}\right) Z$. Since $Z$ is a function of a wide range of parameter values, it may differ between liquidity traps and normal times. In this experiment, we vary the share of the impatient households ($\omega$), and then calculate the corresponding $\omega \left(\frac{\beta - \delta}{\delta}\right) Z$ and the government spending multiplier.

The results are reported in Figure 1. Without credit frictions, the spending multiplier during normal times is slightly below one, which is consistent with previous estimates with separable preferences in consumption and labor (e.g. Dupor and Li, 2015). The normal-times multiplier then declines in the size of the credit friction. The liquidity-trap multiplier is significantly larger in frictionless credit markets but declines rapidly as credit frictions rise. The decline in the spending multiplier when household-side credit frictions are introduced is the main result of our paper. To the extent that one set of agents faces credit constraints and government spending leads to tighter
credit markets (which is validated by these numerical results), the spending multiplier is smaller when credit frictions are present.

The second important finding is that credit frictions matter for the gap between the liquidity-trap and the normal-times multipliers. While in the absence of credit constraints government spending is more effective during liquidity-traps, the gap between both multipliers shrinks as credit frictions rise. With a sufficiently large credit friction, the order is reversed. This finding reflects the observation that the direct and indirect effects of credit frictions on the spending multiplier reinforce each other in liquidity traps but contradict in non-liquidity traps. The analysis in this section thus indicates that ignoring credit constraints is one reason for obtaining larger spending multipliers in liquidity-trap periods.

Third, the liquidity-trap multiplier is considerably larger than one with no credit-constrained households, but drops below one when credit constraints are present. This numerical result supports some of our analytical results even though the analyses are not based on extreme cases.5

The right panel of Figure 1 helps in explaining the changes in the sizes of the multipliers during normal times and liquidity traps. While inflation strongly and positively reacts to spending shocks without credit frictions, the response of inflation declines markedly in liquidity traps compared to normal times. When the response of inflation to a spending shock is zero, the spending multipliers are equal and less than one, in line with condition (54).6 When the response of inflation turns negative, the response of the real interest rate in liquidity traps becomes positive. Then, two effects operate on the liquidity-trap spending multiplier: the rise in the real interest rate and the credit frictions. Both factors reduce consumer spending and the corresponding spending multiplier.

On the other hand, when the monetary authority follows a Taylor rule, the negative response of inflation induces a decline in the nominal interest rate. Importantly, since $\phi_g > 1$, the decline in the nominal interest rate is larger than the decline in the inflation rate, which induces a decline

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5Using a panel of OECD countries, Boehm (forthcoming) find that, at the ZLB, the government spending multiplier is 1.05 at a 4-quarter horizon and 0.93 at an 8-quarter horizon, with standard errors of 0.36 and 0.38, respectively. Therefore, the fact that the ZLB government spending multiplier could be less than one has support in recent empirical analysis.

6Using condition (53), the cut-off value $\omega \left( \frac{\beta - \delta}{\delta} \right) \bar{Z}$ equals $\frac{\omega(1-\rho)\phi_g}{\phi_g(\sigma + \nu c)}$. By solving the simplified version of the model, we obtain $c_g = 0.63$. Then, the cut-off value is roughly 0.017.
in the real interest rate. In this case, there are two opposing effects on consumption: the decline in the real interest rate that is brought about by credit frictions helps in stimulating consumption (indirect effect), while credit frictions directly induce lower consumer spending. The overall effect of credit frictions on consumer spending is modestly negative. These analyses are in line with our analytical results and the discussions that follow them.⁷

The left panel of Figure 2 attempts to numerically support Proposition 1. The figure shows that in the existence of credit-constrained households, the liquidity-trap spending multiplier is less than one even without the extreme case of fully rigid prices. The figure displays the liquidity-trap multiplier when the Calvo parameter (\( \eta \)) is varied between 0.83 and 1. The key insight of this figure is that the liquidity-trap multiplier is clearly less than one even when prices are allowed to be reasonably flexible.⁸

The right panel of Figure 2 corresponds to Proposition 2 and Proposition 3. We show results for a wide range of \( \omega \) between zero and 0.98 and under \( \eta = 0.05 \) (very flexible prices, but not fully flexible) and \( \eta = 0.50 \) (moderately flexible prices). In all cases, the liquidity-trap spending multiplier is less than one. Therefore, to obtain a below-unity spending multiplier, one need not necessarily assume that prices are fully flexible or that all households behave in a Hand-to-Mouth fashion. More reasonable parameters values could lead to the same conclusion.

4.3. Numerical Results- Nonlinear Model

This section presents the results of solving the fully non-linear model (that is outlined in Section 2) under the assumption that the nominal interest rate equals its steady-state value with probability \( \varrho \) and reverts to the flexible interest-rate case (where it follows a Taylor rule) with probability \( 1 - \varrho \).

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⁷Reducing the nominal interest rate following a fall in inflation is obviously possible only when the nominal interest rate is positive. Once the nominal interest rate reaches the ZLB, the economy enters a liquidity trap; as outlined above, fiscal policy then becomes less effective. Additionally, the range of \( \omega \) that we use is such that the spending multipliers of liquidity traps and normal times are non negative. Carlstrom et al. (2014) refer to this scenario as the “normal” case, whereby output rises following a rise in government spending.

⁸Since \( c_1 = 0.63 \), we have \((1 + \nu)(c_1 + g) = 1.24\), which is considerably greater than one. With \( \nu = 0.50 \) and \( g = 0.20 \), this expression becomes less than one if and only if \( c_1 \) is less than 0.47, which is a very low consumption-output ratio. In addition, the larger \( \nu \) or \( g \) are, the more likely \((1 + \nu)(c_1 + g)\) to be greater than one, suggesting that the multiplier in (58) is very likely to be less than one.
Unlike in Section 3, at this stage, we do not force the probability that the nominal interest rate is fixed \((\varrho)\) to equal the persistence of government spending \((\rho)\). Instead, we let \(\varrho\) be sufficiently large in order to minimize the response of \(R_t\) to spending shocks.\(^9\) This way, we can focus on the effects of credit frictions on the effectiveness of fiscal policy without being concerned about the degree to which monetary policy is accommodative, on the one hand, or determinacy, on the other. In the online appendix, we also show the results when \(\varrho = \rho\).

### 4.3.1. Deterministic Steady State

Table A.2 presents the steady-state values that we obtain by solving the non-linear version of the model with and without credit frictions. Since one can think of the model without credit constraints as having one type of households, consumption, labor and other variables are the same for both types of agents \((\overline{\pi} = \overline{x} = \overline{r} \text{ for every variable } x)\), and thus we report the value of \(\overline{\pi}\). Furthermore, with one type of households, there is no borrowing or lending between households, hence \(\overline{\gamma} = \overline{d} = 0\).

We briefly comment on some of the steady-state values. First, since the total labor supply by each type of household is the same (by design, as discussed in Subsection 4.1), the steady-state level of output is the same for both models (with and without credit frictions). As a result, the steady-state level of government spending is the same for both models. Second, consumption of the impatient households is clearly lower than consumption of the patient households, which reflects the effects of the credit constraints. Third, the value of \(\overline{d}\) suggests that the steady-state deposits-output ratio is roughly 46%. Overall, this estimate is in line with the average ratio of total checkable and saving deposits to GDP in the U.S. (which is 34% over the period 1962:Q2-2016:Q4, for which data are available). Fourth, \(\overline{b}_{\text{Share}}\) is very similar to the share of credit to households of total credit (namely, credit to households as a percentage of the sum of credit to households and the government). In U.S. data, the average of this ratio over the period 1960:Q2-2016:Q4 is 52%.\(^{10}\)

We thus view the model-implied figures as well capturing some basic U.S. empirical regularities regarding credit and deposits even though we do not explicitly target them in the model.

Finally, given the parameter values and the steady-state values \(\overline{\gamma}\) and \(\overline{\pi}\), the benchmark value of \(M\) is 0.023, which supports our assumption in Subsection 3.1. In addition, \(M\) remains positive for all values of \(\kappa\) that satisfy determinacy and for all values of \(\omega\).

### 4.3.2. Impulse Responses

This section discusses impulse response functions following a positive 1% government spending shock. With no credit frictions. A shock to government spending raises output and inflation, the real interest rate \((r)\) drops (and so does the expected real interest rate), which in turn leads to a surge in consumer spending and further raises the response of output (Figure 3).

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\(^9\)This experiment corresponds to one scenario that Christiano et al. (2011) have discussed. Essentially, this scenario suggests that monetary policy is very accommodative of an expansionary fiscal policy. In addition, the Federal Funds Rate remained virtually constant for nearly 29 quarters (December 2008-December 2015) during and in the aftermath of the Financial Crisis. Letting \(T\) be the duration of the constant interest rate policy, the probability of a constant interest rate is then \(\varrho = (T - 1)/T\), which is roughly 0.97. We use this value in our benchmark analysis.

\(^{10}\)Specifically, we use the sum of Total Checkable Deposits (#TCDSL) and Savings Deposits - Total (#SAVINGSL) and GDP Gross Domestic Product (#GDP). For credit, we use Total Credit to Households and Non-Profit Institutions Serving Households, Adjusted for Breaks (#QUSHAM770A) and Total Credit to General Government, Adjusted for Breaks (#QUSGAM770A). All series are quarterly and they are available at the FRED database of the Federal Reserve Bank of St. Louis.
Matters differ when credit frictions are introduced. While output rises in response to the government spending shock, total consumption declines. The decline in total consumption is a result of four factors. First, inflation slightly declines on impact, which in turn induces a slight rise in the real interest rate. This channel in isolation negatively affects consumption of both types of households. For the impatient households, even though the nominal cost of borrowing ($R_t$) barely changes, the real cost of borrowing rises, which further depresses their borrowing and consumption. This effect basically corresponds to the “indirect effect” of credit constraints on consumer spending that has been discussed in Section 3. Second, the tighter credit constraint induces a bigger drop in consumption of the impatient agents and reduces total consumption (the “direct effect” of credit constraints). Third, the rise in government spending induces a rise in government borrowing ($b_{G,t}$), which comes at the expense of private borrowing. In turn, this effect tightens the credit constraint and limits the ability to smooth consumption. Fourth, since taxes of both types of households rise on impact, consumption of both types of households is negatively affected.

A natural question is what makes the credit constraint become tighter following the rise in government spending? Due to the binding credit constraint, the starting point of consumption of the constrained agents is lower than in a model without credit constraints. Therefore, these agents would like to supply more labor (a wealth effect), which puts downward pressure on their real wages (these agents would like also to work more in an attempt to make their credit constraints less tight). On the other hand, with the credit constraint, total demand ($c_t + g_t$) is smaller than in a model with no constraints. Since output is demand-driven, firms meet the reduced demand for their products by producing less than in the credit-frictionless model. That leads to a reduction in the demand for both types of labor, places downward pressures on the real wages of the patient agents and to further downward pressure on the real wages of the impatient agents.

For the impatient households, two opposing effects operate on their labor ($n_{I,t}$): a higher labor supply and a lower labor demand. In sum, $n_{I,t}$ rises following a rise in government spending, but by less than the corresponding rise in $n_{I,t}$ when the model abstracts from credit constraints. As for the labor of the patient households ($n_{P,t}$), there is only (or primarily) a labor demand effect. Therefore, their labor rises by less than the rise in the labor of the impatient households. These effects combined induce a fall in the real wage of the impatient households and a modest fall in the real wage of the patient households. The decline in the real wage of the impatient households, in turn, limits their resources, which has two implications: first, the impatient households will consume less unless they borrow more and/or sell some of their real estate holdings. In the model’s simulations, they resort to both methods, which leads to a decline in the value of their assets and a rise in the desire to borrow. The impatient households attempt to borrow more at the same time that their real estate value has declined. Therefore, as implied by condition (14), their borrowing falls and the credit constraint tightens (notice that the largest rise in $\lambda_{I,t}$ occurs when $h_{I,t}$ reaches its lowest point).

This mechanism is amplified by the rise in taxes (which reduces the disposable income of all

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11 Bronery et al. (2018) find that, for a panel of advanced economies, government spending crowds out private borrowing, and by extension private consumption and investment, unless debt is issued to foreigners. In particular, the study finds that the fiscal multiplier is smaller than one when the foreign share of public debt is low.

12 Rossi and Trucchi (2016) show that in the presence of credit constraints, households raise their supply of labor in an attempt to loosen the credit constraints. Our quantitative results align with their analytical and empirical findings.
households) and borrowing by the government (which limits the amount of funds that is available for private borrowing). The higher taxes, the smaller the rise in the disposable income of the patient households and the higher lending to the government limit the amount of their deposits. In turn, the supply of funds to the impatient households is reduced, exactly at the time when the impatient households would like to borrow more. That causes the credit constraint to tighten as well.

The lower real wages (particulary of the impatient households) induce a decline in the real marginal cost, which triggers a decline in the inflation rate. Then, the real interest rate rises on impact, triggering a crowding-out effect on consumption of both types of households. In this respect, the credit constraints do not allow the expected inflation channel to generate a large liquidity-trap.
spending multiplier, but rather leads to the opposite outcome.\footnote{This result is in line with Faia and Monacelli (2007), who show that the response of inflation to a TFP shock turns from positive when no credit frictions are present to negative when these frictions are introduced. In addition, the decline in the real marginal cost with the rising output under credit frictions implies a procyclical price markup as opposed to a countercyclical price markup in the absence of credit frictions (which is crucial for obtaining large spending multipliers). Empirically, Nekarda and Ramey (2013) show that the price markups are either procyclical or acyclical conditional on demand shocks. Our results in the model with credit constraints are consistent with their empirical findings.}

Finally, to some extent, our results (in particular, the decline in total consumption) are driven by the type of the utility function that is used. As the government increases its spending, the resources that are available to the private sector fall, thereby reducing private wealth. Households respond by raising their labor supply. Since preferences are separable in consumption and labor, the labor supply conditions are given by $\sigma c_i, t = \bar{w}_i,t - \nu n_{i,t}$, with $i = I, P$. Therefore, an increase in labor would reduce consumption unless real wages sufficiently rise. In Figure 3, labor supply of the impatient households rises significantly, thus driving down their real wage and consumption. However, as we show in what follows, once labor and consumption become complementary, consumption rises when households work more, and thus the government spending multiplier remains above one even with credit constraints.\footnote{See Monacelli and Perotti (2008) for more discussion. Furthermore, with no credit constraints, labor supply does not rise as much and labor demand rises. Therefore, real wages rise, and consumption rises together with labor (namely, in that case, $\bar{w}_{i,t} > \nu n_{i,t}$).}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{The impact spending multiplier in a liquidity trap.}
\end{figure}

### 4.3.3. Implied Impact Government Spending Multiplier

We use the impulse response functions to calculate the implied impact spending multiplier, which is the increase in the level of output $k$ periods ahead ($\Delta y_{t+k}$) in response to a rise in government spending at the current period ($\Delta g_t$). Namely, the impact multiplier is given by $\Delta y_{t+k}/\Delta g_t$. The impact liquidity-trap spending multiplier with credit frictions is significantly smaller than the multiplier in the model without these frictions both on impact and over longer horizons (Figure 4). In particular, on impact these multipliers are 0.68 and 1.53, respectively.

### 4.4. Relation to the Literature

Having discussed the results of this paper and the mechanisms in play, we now comment on the relation between our paper and previous work. As discussed in the introduction, Carrillo and Poilly (2013) find that financial frictions, that are introduced as in the financial accelerator model

13This result is in line with Faia and Monacelli (2007), who show that the response of inflation to a TFP shock turns from positive when no credit frictions are present to negative when these frictions are introduced. In addition, the decline in the real marginal cost with the rising output under credit frictions implies a procyclical price markup as opposed to a countercyclical price markup in the absence of credit frictions (which is crucial for obtaining large spending multipliers). Empirically, Nekarda and Ramey (2013) show that the price markups are either procyclical or acyclical conditional on demand shocks. Our results in the model with credit constraints are consistent with their empirical findings.

14See Monacelli and Perotti (2008) for more discussion. Furthermore, with no credit constraints, labor supply does not rise as much and labor demand rises. Therefore, real wages rise, and consumption rises together with labor (namely, in that case, $\bar{w}_{i,t} > \nu n_{i,t}$).
of Bernanke et al. (1999), raise the spending multiplier at the ZLB, with investment playing a major role in driving these results. We have few comments on the differences between our study and Carrillo and Poilly (2013). First, in the latter study, households could raise their consumption freely as the agency cost was imposed on entrepreneurs. Indeed, consumption in their study rises by more in the model with financial frictions than in the model with no such frictions, suggesting that investment is not the only driver of their results, and thus not the only reason for the differences between our findings and theirs. In our model, one set of households cannot freely raise their consumption, which weakens the effectiveness of fiscal policy.

Second, the effects of changes in housing values on consumption have been recently documented in the literature. Mian and Sufi (2012) study the effects of the “Cash for Clunkers” program in 2009, whereby the government provided payments to dealers for each old less fuel-efficient car that is traded in by a consumer who purchased a new more fuel-efficient car. The authors find no effects on employment in cities with high exposure to the program. In addition, by looking at a one year span (July 2009- June 2010), the program did not have a meaningful impact on car purchases. Mian et al. (2013) document that from 2006 to 2009, consumer spending has declined substantially more in U.S. counties with a large decline in housing net worth and that the “household balance sheet” channel can explain a large fraction of the decline in consumer spending. Mian and Sufi (2014) show that a decline in housing net worth could suppress consumer spending either because of a direct wealth effect or through tighter borrowing constraints that result from the fall in collateral values. In addition, they show that deterioration in household balance sheets played a major role in the sharp decline in U.S. employment between 2007 and 2009. Pistaferri (2016) concludes that financial frictions were the trigger of the sharp decline and the subsequent weakness of consumption in the aftermath of the Financial Crisis. A decline in the housing prices impacted consumer spending through multiple channels such as a direct wealth effect, a rise in borrowing constraints (as financial intermediaries were less willing to lend against lower and less certain collateral values) and the “leverage” effect (whereby the decline in housing values increased the debt-asset ratio, which in turn required sharp adjustments in consumption).

In relation to these findings, our analysis evolves around the behavior of consumer spending where real estate serves as collateral as well as being a component of the assets of both types of households. We then discuss how changes in the value of collateral affect the tightness of credit constraints. This channel is absent from Carrillo and Poilly (2013), and our paper studies it (which is particularly important given that consumption constitutes nearly 70% of the U.S. GDP). In addition, the numerical results of this paper illustrate that, following a rise in government spending, the rise in the labor input is smaller when credit constraints are present than otherwise.

Third, since Carrillo and Poilly (2013) abstract from demand-side credit frictions, the rise in government spending raises inflation, and thus leads to a decline in the real interest rate (and relatively cheaper credit). However, downturn periods are not typically associated with relatively cheaper credit or higher inflation rates. And, as Dupor and Li (2015) empirically show, the expected inflation channel might have been too small (or nonexistent) during the American Recovery and Reinvestment Act episode to induce large spending multipliers.15 Our study, on the other hand, captures these features.

15Using OECD data, Boehm (forthcoming) find that the rise in government spending had no statistically significant impact on the percentage change of the prices of the consumption goods (or investment goods) at the ZLB.
Our model, thus, accounts for certain aspects that have been found to be important for consumption and employment behavior during the ZLB episode in the United States. These aspects are not present in Carrillo and Poilly (2013), which could explain some of the differences in the results.

5. Sensitivity Analysis

There are two important factors behind our main findings: the behavior of inflation and the response of the shadow value of the credit constraint to spending shocks. We present analyses that are meant to clarify the roles of each factor. We then discuss other robustness analyses. All the results of this section are obtained by solving the full model.

5.1. Alternative Parameter Values

For liquidity traps, we experiment on key parameters that pertain to the constrained households: the loan-to-value ratio ($\tau$), the discount factor of the impatient households ($\delta$) and the share of the impatient households in the population ($\omega$). In addition, we vary the persistence parameter of government spending ($\rho$) to examine the extent to which the spending multiplier (hence, the coefficient $Z_{LT}$) is endogenous to the persistence of fiscal policy.

Figure 5: The liquidity trap-spending multiplier for various values of model parameters. Notes: with the exception of $\rho$, the benchmark value of each parameter lies in the middle of the range. To satisfy determinacy, $\rho$ cannot not exceed 0.82. Each time, we change the parameter of interest while keeping other parameters at their benchmark values. The dashed horizontal line indicates the spending multiplier with no frictions.

Figure 5 shows that the main result of this paper, that credit constraints induce smaller spending multipliers, mostly holds for a wide range of parameter values (therefore, in almost all cases that we consider here, the sign of $Z_{LT}$ is positive). Some of the effects of changing these parameter values on the spending multiplier are straightforward: a larger share of impatient households ($\omega$) or a smaller discount factor of these households ($\delta$) reduce the multiplier. As government spending become less persistent, the spending multiplier approaches 1 (with and without credit constraints).

The government spending multiplier does not appear to be monotonic in the the loan-to-value ratio ($\tau$), albeit it is clearly high when $\tau$ is relatively large (above 1 in this case). With $\tau = 1$, the value of $Z_{LT}$ is very large, and the spending multiplier ends up being small. On the other hand, with $\tau = 1.2$, the spending multiplier with credit constraints is larger than the spending multiplier that abstracts from these constraints (namely, $Z_{LT}^c < 0$). This is the only case where we find that
the spending multiplier is larger when credit constraints are present. The value of collateral in this case is sufficiently large that the credit constraint sufficiently loosens to cause a larger spending multiplier. Notice, however, that this value of \( \tau \) is 1.5 times the benchmark value of this parameter, and thus very large loan-to-value ratio is need to alter the main finding of the paper.\(^{16}\)

5.2. The Roles of Inflation and Tightness of the Credit Constraint

To shed some light on the role of inflation and the tightness of the credit constraint in explaining the results that we obtain in Figure 3, this subsection presents analyses when the tightness of the credit constraint does not change following a rise in government spending (\( \bar{\lambda}_{I,t} = 0 \)). This may occur if collateral happens to move by exactly the amount that is needed to meet the rise in borrowing by the impatient households. The analyses are in support of the discussion in Subsection 3.3.

Panel A of Figure 6 shows the spending multiplier and the response of inflation to spending shocks (\( A_\pi \)) for various values of \( \omega \) under the benchmark value of the price rigidity parameter (\( \kappa \)). Both the spending multiplier and \( A_\pi \) fall when \( \omega \) rises. According to Panel B, the falls in both are bigger when prices are more flexible (higher \( \kappa \)). These findings are in line with conditions (55)-(56). A weaker demand (due to credit constraints) is associated with a weaker response by the inflation rate to a government spending shock and a smaller spending multiplier. The more flexible prices are, the more responsive the inflation rate is. Panel C presents the spending multiplier as a function of the price rigidity parameter (\( \kappa \)). For the same degree of price rigidity, the spending multiplier with credit constraints (but constant \( \lambda_{I,t} \)) is smaller, but the two multipliers are very

\(^{16}\)Also, changing \( \sigma \) within the range 1-3 and \( \nu \) within the range 0-1 does not affect the main conclusion of the paper.
similar when price rigidity is very high (low $\kappa$). These three panels suggest that for the spending multiplier with credit constraints to be smaller, credit constraints should not necessarily become tighter when government spending rises, and that the role of inflation could be very important.

In Figure 4, we find that the spending multiplier with credit constraints is less than one, while with a constant $\lambda_{I,t}$, the spending multiplier is larger than one. Therefore, we may attribute the gap between the spending multipliers in Panel A of Figure 6 and Figure 4 to the fact that the credit constraint becomes tighter. In particular, the spending multiplier with no credit constraints is larger than the multiplier with credit constraints and a constant $\lambda_{I,t}$, which in turn exceeds the multiplier with credit constraints and a variable $\lambda_{I,t}$. As such, while the expected inflation channel is important, it is not the only factor in generating the results that are presented in the benchmark analysis. In the extreme case of fully rigid prices, only tighter credit constraints can make the spending multiplier smaller. On the other hand, with fully flexible prices, credit constraints do not affect the spending multiplier. In any intermediate case, both the tightness of the credit constraint and the inflation rate matter for the size of the multiplier.

Since the spending multiplier with $\lambda_{I,t} = 0$ is smaller than the spending multiplier with no credit constraints, a natural question is by how much should the credit constraint become looser so that the model with credit constraints yields at least the same spending multiplier as in the model with no credit constraints. Namely, what decline in $\lambda_{I,t}$ satisfies $A_y(\omega > 0) = A_y(\omega = 0)$? The answer to this question is shown in Panel D of Figure 6 for various values of $\kappa$. When prices are fully rigid ($\kappa = 0$), inflation is zero and the credit constraint should not fall to equate the multipliers with and without frictions. However, as prices become more flexible, the credit constraints should become looser to equate the spending multipliers under both scenarios. An important insight that comes out of this analysis is the following: if the tightness of the credit constraint falls by less than indicated by this panel, then the spending multiplier with credit frictions will still be smaller than the spending multiplier without frictions. Therefore, the decline in $\lambda_{I,t}$ should exceed a certain threshold so that credit frictions generate larger spending multipliers.

5.3. The Role of Taxes and Crowding-Out of Private Borrowing

This subsection studies two alternative cases. First, a balanced budget, where the government relies on taxes only to finance its spending ($b_{G,t} = 0$). We consider two scenarios for the balanced-budget multiplier: taxes are levied on both types of households, $g_t = \omega T_{I,t} + (1 - \omega) T_{P,t}$, and taxes are levied only on the patient households, $g_t = (1 - \omega) T_{P,t}$. The results are summarized in Figure A.1. In this case, the credit constraint becomes looser, and consumption of the impatient households ($c_{I,t}$) rises. The largest increase in $c_{I,t}$ occurs when taxes are imposed only on the patient agents: there is no crowding out of private borrowing and the impatient agents do not bear the burden of taxes. A rise in the labor income of the impatient agents, thus, induces a strong rise in their consumption. On the other hand, total consumption and output are held back by the bigger decline in consumption of the patient agents, which reflects a stronger rise in their taxes when they bear all the burden of financing the rise in government spending. Overall, this scenario brings the spending multiplier closer to its value in the model with no credit constraints.

These analyses shed light on the role of the crowding out of private borrowing in reducing the effectiveness of fiscal policy. In addition, they suggest that government spending will be more effective if the constrained agents pay less taxes. However, since levying more taxes on the unconstrained agents (who constitute a larger proportion of the population) hampers their consumption
and the credit constraint does not loosen sufficiently, the spending multiplier remains smaller than the spending multiplier without credit constraints.

Second, we re-do the analysis under the assumption that taxes do not rise on impact and that the government finances the rise in spending by issuing bonds. Differently from the previous case, there is a slight increase in the consumption of the patient households, but total consumption is held back by the tighter credit constraint: the additional government borrowing crowds out private borrowing and consumption of the impatient agents. In fact, one can attribute almost the entire decline in total consumption in this case to the credit constraint-induced decline in the consumption of the impatient households.

By comparing the results in this section to Figure 3, we learn that the assumption regarding the method of financing the rise government spending has implications for the behavior of consumption of both types of agents (hence, total consumption and output). Therefore, the behavior of taxes and the crowding-out of private borrowing can explain part of the findings in the benchmark analysis when credit constraints are present.

5.4. The Role of Preferences

In this subsection, we briefly discuss the role of preferences in generating the results that we obtain in the benchmark analysis. In the benchmark analysis with separable preferences, consumption has declined in the presence of credit constraints. In this subsection, we use non-separable preferences as in Christiano et al. (2011), which are given by:

\[
    u(c_{i,t}, n_{i,t}) = \frac{[c_{i,t}^{\gamma}(1-n_{i,t})^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma}
\]

with \(i = I, P\). With these preferences, there is a complementarity between labor and consumption: the marginal utility of consumption rises with labor, thus generating a spending multiplier above one (even during normal times). Indeed, Christiano et al. (2011) find that the spending multiplier with these preferences is 3.7 during liquidity traps.

In Figure A.2, we present the results using these preferences. Without credit constraints, our impact multiplier is exactly as in Christiano et al. (2011). When credit constraints are introduced, the response of consumption of both types of agents is smaller, and therefore the spending multiplier is smaller. In this case, however, consumption of the impatient households rises but by less than it would in the absence of credit constraints. The main takeaway of this exercise is that the assumption regarding the form of preferences matters for the size of the multiplier, and thus we can partly attribute the less-than-unity spending multiplier in the benchmark analysis to the preferences that we use in that section.\(^{17}\)

\(^{17}\)In the online appendix, we provide the following robustness analyses. 1) The nominal interest rate equals its steady-state value with probability \(\varrho = 0.8\) and reverts to the flexible interest-rate case (where it follows a Taylor rule) with probability \(1 - \varrho\). In other words, we consider an intermediate case between what liquidity trap and normal times. 2) The nominal interest rate equals one \((R_t = 1)\) with probability \(\varrho = 0.8\) and reverts to the flexible interest-rate case (where it follows a Taylor rule) with probability \(1 - \varrho\). 3) The ZLB constraint only occasionally binds. 4) We study the spending multiplier in a model with investment in physical capital. In all cases, we obtain similar results regarding the effects of credit constraints on the government spending multiplier.
5.5. Normal Times

Figure A.3 shows the behavior of the main variables in normal times (where the nominal interest rate follows a Taylor-type rule). The first observation is that the effect of credit constraints on the response of output to a spending shock is modest. Second, the binding credit constraint reduces consumption of the impatient households by more than in the model without frictions. This amounts to the direct effect of the credit constraint on the consumption of these agents. Third, since the response of inflation to a government spending shock is weaker when credit constraints are present, the rise in the nominal interest rate and the corresponding rise in the real interest rate are smaller. This channel in itself limits the crowding-out effect on consumption of both types of households, particularly consumption of the patient households; it falls only slightly on impact before rising again. Consequently, the effect of credit constraints on total consumption is minor.

These findings are consistent with the analysis that we have made in Section 3. Unlike liquidity traps, credit constraints have two opposing effects on total consumption (hence, the government spending multiplier) in normal times. The direct effect reduces consumption while the indirect effect increases consumption relative to a model with no credit constraints, a reminiscent of Proposition 3 and the discussion that follows. In addition, the comparison between Figure 3 and Figure A.3 suggests that the liquidity-trap multiplier is larger than the normal-times multiplier when credit constraints are absent, but the liquidity-trap multiplier is smaller when credit constraints are introduced. This constitutes another important finding of this paper: with credit constraints, government spending is not necessarily more effective in liquidity traps than in normal times.

6. Conclusions

In this paper, we study the implications of credit frictions for the government spending multiplier within a quantitative business cycle model. The model embeds credit constraints on one set of households and finds that the government spending multiplier during liquidity traps is smaller than in a model that abstracts from these constraints. This result holds even if the rise in government spending does not make credit constraints tighter. The latter reflects a weaker response of inflation to spending shocks that, in turn, induces a smaller decline (or even a rise) in the real interest rate, and thus limits the rise (or leads to a decline) in consumer spending. In this respect, credit constraints weaken the “expected inflation channel”, which has been crucial for obtaining large liquidity-trap spending multipliers in quantitative models. It is then shown that the rise in government spending crowds out private borrowing and induces tighter credit constraints that further reduce the spending multiplier.

In addition, in the absence of credit constraints, the spending multiplier of liquidity-trap periods is markedly larger than the spending multiplier during normal times (during which the nominal interest rate is free to adjust). However, introducing credit frictions could reverse this result. Therefore, our analyses not only cast doubts about government spending being more effective because of credit frictions but that this does not occur in liquidity traps. The paper also demonstrates that, in the presence of credit frictions, the spending multiplier during liquidity traps could be less than one.
References


### A. Tables and Figures

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<tr>
<th>Variable</th>
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Table A.1: Empirical results- Generalized Method of Moments (GMM) and Ordinary Least Squares (OLS) estimation of condition (61). Standard errors in parentheses. * Denotes significance at the 10% level. ** Denotes significance at the 5% level. *** Denotes significance at the 1% level. We use the following quarterly data series: 1) Federal government current tax receipts, Billions of Dollars (#W006RC1Q027SBEA) for $T_t$, 2) Federal Debt: Total Public Debt, Millions of Dollars (#GFDEBTN) for $b_{G,t}$, 3) Federal government total expenditures, Billions of Dollars (#W019RCQ027SBEA) for $g_t$. All variables were divided by the size of the working-age population. For GMM, we use four lags of each variable as instruments. $J$-Test tests the hypothesis that the over-identifying restrictions are satisfied. Sample period: 1967:Q1-2016:Q4. Source: the FRED database of the Federal Reserve Bank of St. Louis.

<table>
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Table A.2: Deterministic Steady-State Values of the non-linear models with credit frictions and without credit frictions. Note: $\overline{R}_I = \overline{R}$ and $\overline{R}_D = \overline{R}$. 

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Figure A.1: Impulse responses to a positive 1% government spending shock with a constant nominal interest rate, expressed as percentage deviations from the deterministic steady state. Notes: 1) “No Frictions” - the benchmark model with no credit frictions. 2) “Balanced Budget, $T_I&T_P$” - the model with credit frictions, balanced budget, and taxes on both types of agents. 3) “Balanced Budget, $T_P$ Only” - the model with credit frictions, balanced budget and no taxes on the impatient households. “Constant Taxes, $T_I&T_P$” - the model with credit frictions, and constant taxes on both types of agents ($T_{B,t} = T_B$, $T_{L,t} = T_L$).

Figure A.2: Impulse responses to a positive 1% government spending shock in a liquidity trap, expressed as percentage deviations from the deterministic steady state. Notes: the time unit is a quarter. Non-separable preferences. Following Christiano et al. (2011), we set $\sigma = 2$ and $\gamma = 0.29$.

Figure A.3: Impulse responses to a positive 1% government spending shock in normal times, expressed as percentage deviations from the deterministic steady state. Note: the coefficients of the nominal interest-rate rule: $\phi_\pi = 1.5, \phi_y = 0.0$. 


B. The Log-Linearized Model

\[ \bar{R}_t - \mathbb{E}_t \bar{\pi}_{t+1} = \sigma(\mathbb{E}_t c_{P,t+1} - \bar{c}_P) \]  
(\text{B.1})

\[ \nu \bar{c}_P + \sigma \bar{c}_P = \bar{w}_P \]  
(\text{B.2})

\[ q c_P^{-\sigma} (\sigma c_{P,t} - \bar{q}_t) = \frac{\psi_p}{h_P} h_{P,t} + \beta q c_P^{-\sigma} \mathbb{E}_t (\sigma c_{P,t+1} - \bar{q}_{t+1}) \]  
(\text{B.3})

\[ \bar{R}_t - \mathbb{E}_t \bar{\pi}_{t+1} = \sigma(\mathbb{E}_t c_{I,t+1} - \bar{c}_I) - \frac{\bar{\mu}}{1 - \bar{\mu}} \]  
(\text{B.4})

\[ \nu \bar{c}_I + \sigma \bar{c}_I = \bar{w}_I \]  
(\text{B.5})

\[ q c_I^{-\sigma} (\sigma c_{I,t} - \bar{q}_t) = \psi_I \bar{h}_{I,t} + \delta q c_I^{-\sigma} \mathbb{E}_t [(1 + \tau \bar{\mu})(\sigma c_{I,t+1} - \bar{q}_{t+1}) - \tau \bar{\mu} \bar{\mu}_{t+1}] \]  
(\text{B.6})

\[ \bar{c}_I c_{I,t} + \bar{q} \bar{h}_I (\bar{h}_{I,t} - \bar{h}_{I,t-1}) + \frac{R b_I}{\bar{\pi}} (\bar{R}_{t-1} + \bar{b}_{I,t-1} - \bar{n}_t) + \bar{T}_t \bar{T}_I = \bar{w}_I \bar{m}_I (\bar{w}_{I,t} + \bar{n}_I) + \bar{b}_I \bar{b}_I \]  
(\text{B.7})

\[ \bar{b}_{I,t} = \bar{\bar{q}}_t + \bar{h}_{I,t-1} \]  
(\text{B.8})

\[ \bar{m}_I + \alpha \bar{n}_{I,t} - \alpha \bar{n}_{P,t} = \bar{w}_P \]  
(\text{B.9})

\[ \bar{m}_I + (\alpha - 1) \bar{n}_{I,t} + (1 - \alpha) \bar{n}_{P,t} = \bar{w}_I \]  
(\text{B.10})

\[ \bar{\pi}_t = \beta \mathbb{E}_t \bar{\pi}_{t+1} + \kappa \bar{m}_I \]  
(\text{B.11})

with \( \kappa = \frac{(1-\eta)(1-\beta \eta)}{\eta} \). The parameter \( \kappa \) is zero with fully rigid prices (\( \eta = 1 \)) and approaches infinity when prices are fully flexible (\( \eta = 0 \)).

\[ g = \frac{\bar{g}}{\bar{\pi}}, \ c_i = \frac{c}{\bar{\pi}} \]  
(\text{B.12})

with \( g = \frac{\bar{g}}{\bar{\pi}}, \ c_i = \frac{c}{\bar{\pi}} \) and \( c_P = \frac{c}{\bar{\pi}} \) being the steady-state government spending-output ratio, the consumption-output ratio of the impatient households and the consumption-output ratio of the patient households, respectively.

\[ \bar{g}_t = \rho \bar{g}_{t-1} + u_t \]  
(\text{B.13})

\[ \bar{T}_P = \rho \bar{c}_P + \rho \bar{g}_t \]  
(\text{B.14})

\[ \bar{T}_I = \rho \bar{c}_I + \rho \bar{g}_t \]  
(\text{B.15})

\[ \bar{T}_{P,t} = \phi_{\pi} \bar{\pi}_t + \phi_{\bar{g}} \bar{g}_t \]  
(\text{B.16})

\[ \omega \bar{h}_{I,t} + (1 - \omega) \bar{h}_PH_{P,t} = 0 \]  
(\text{B.17})

\[ \bar{b}_{P,t} = \bar{b}_{G,t} \]  
(\text{B.18})

\[ \bar{d}_{I,t} = \bar{b}_{I,t} \]  
(\text{B.19})

\[ \bar{g}_t = \alpha \bar{m}_{I,t} + (1 - \alpha) \bar{n}_{P,t} \]  
(\text{B.20})

Also, multiplying condition (\text{B.9}) by \((1 - \alpha)\) and multiplying condition (\text{B.10}) by \(\alpha\), and then combining the resultant conditions give \( \bar{m}_I = \bar{\bar{q}}_t \) with \( \bar{\bar{q}}_t = \alpha \bar{w}_{I,t} + (1 - \alpha) \bar{w}_{P,t} \).

Notice that to obtain conditions (\text{B.13}), (\text{B.14})-(\text{B.15}) and (\text{B.17}), we use the fact that for every
variable $x_t$ and for a sufficiently small $\tilde{x}_t$, we have $\ln(\frac{x}{\tilde{x}}) \equiv \ln(1 + \tilde{x}_t) \approx \tilde{x}_t$. In addition, since we linearize around a zero steady-state inflation rate (in which $\bar{\Delta} = 1$), the log-deviation of the price dispersion is zero ($\bar{\Delta}_t = 0$). In particular, the price dispersion is a second-order phenomenon in this case, and can be ignored up to first-order approximation. For this reason, and as is standard in this class of models, it does not appear in our log-linearized framework.

C. The Liquidity-Trap Spending Multiplier With a Constant $\lambda_{I,t}$

When $\lambda_{I,t} = 0$, the Euler equation of the impatient households is given by:

$$E_t \tilde{\pi}_{t+1} = \sigma \left( \frac{\beta}{\delta} \tilde{c}_{I,t} - E_t \tilde{c}_{I,t+1} \right)$$  \hspace{1cm} (C.1)

We then obtain the following set of equations:

$$\varrho A_\pi = \sigma (1 - \varrho) A_P$$  \hspace{1cm} (C.2)

$$\varrho A_\pi = \sigma \left( \frac{\beta}{\delta} - \varrho \right) A_I$$  \hspace{1cm} (C.3)

$$(1 - \beta \varrho) A_\pi = \kappa [\nu A_y + \omega \sigma A_I + (1 - \omega) \sigma A_P]$$  \hspace{1cm} (C.4)

$$A_y = \omega c_{I} A_I + (1 - \omega) c_{P} A_P + g$$  \hspace{1cm} (C.5)

which is a set of 4 equations with 4 unknowns ($A_y, A_I, A_P, A_\pi$). The solution of this set of equations gives conditions (55) and (56) in the text, dividing condition (C.3) by condition (C.2) gives condition (57) in the text. Notice also that the budget constraint of the government is not part of this set of equations; therefore, these results do not rely on a specific method of financing the rise in government spending. In addition, the results do not rely on any approximations.

Once $A_I$ is found from this system, one can pin down the value of $Z_{LT}$: $\lambda_{I,t} = 0$ implies $\mu_t = \sigma c_{I,t}$. Therefore $Z_{LT} = \sigma A_I$. Furthermore, since $A_\pi$ is positive for any $\omega$ (see condition (56)), $A_I$ is positive too. Therefore, $Z_{LT} > 0$. This observation is consistent with a smaller spending multiplier in the presence of credit frictions.

To obtain Panels A, B and C in Figure 6, we solve the full model where $\lambda_{I,t}$ is set at its steady state value (i.e. $\lambda_{I,t} = \bar{\lambda}_I$ for all $t$). To calculate the required fall in $\lambda_{I,t}$ that is needed to equate the spending multipliers with and without frictions (Panel D), one may think in the following way: when credit frictions do not affect the spending multiplier, we have $\mu_t = \bar{\mu}$ (which implies $Z_{LT} = 0$). Then, we obtain $\lambda_{I,t} = \bar{\mu} c_{I,t}^{-\sigma}$. Since $c_{I,t}$ rises with $\mu_t = \bar{\mu}$, $\lambda_{I,t}$ falls. This fall is then reported in Panel D of Figure 6.

Finally, by combining conditions (C.2) and (C.3), we obtain condition (57) in the text. The response of consumption of the constrained households is weaker than the response of the consumption of the unconstrained households. As such, the response of total consumption in the model with credit-constrained households is smaller than the response of total consumption in the model without credit constrained households. The smaller spending multiplier then follows. In this setup, a weaker response by consumption (of both types of agents), is associated with a weaker response of inflation to a government spending shock (lower $A_\pi$). Since the expected inflation rate is given by $E_t \tilde{\pi}_{t+1} = \varrho \tilde{\pi}_t$ and $\varrho$ is positive, a weaker response by the inflation rate implies a weaker response by the expected inflation rate.

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