A Fiscal Perspective of Nominal GDP Targeting

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Abstract

This paper studies fiscal policy in a model with nominal GDP (NGDP) targeting. We find that adopting this regime makes standard fiscal policy rules, whereby distortionary tax rates are designed to respond to real economic activity, dependent on the behavior of inflation instead. In particular, if inflation declines following a negative demand shock, then the fiscal policy rules with NGDP targeting call for higher tax rates concurrently with the fall in output. Second, we show that the government spending multiplier and the tax multipliers with NGDP targeting are lower than the multipliers under inflation targeting and a Taylor rule. In this respect, the degree to which monetary policy can manage the economy to achieve the nominal GDP target is crucial for the results.

Keywords: Fiscal Policy; Nominal GDP Targeting; Inflation Targeting; Government Spending Multiplier; Tax Multipliers.

JEL Classification: E31; E52; E58; E62; H30.

1. Introduction

In recent years, there has been a growing interest in studying nominal gross domestic product (NGDP) targeting. Most discussion has been about monetary policy. This paper studies fiscal policy in a model with NGDP targeting. We characterize optimal fiscal policy and compare it to a myriad of monetary policy rules and calculate the government spending and tax multipliers in this environment. Our results do not indicate any clear advantage of adopting nominal GDP targeting from the perspective of fiscal policy. Instead, the analyses point to potential issues that are associated with adopting this regime.

The study uses a standard dynamic stochastic general equilibrium (DSGE) model with households, firms, a central government and a monetary authority. The government finances its expenditures via bond issuance and taxes on labor income, capital income, consumption and profits. Taxation is represented by tax rate rules that respond to real output and debt

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as in Leeper et al. (2010), Leeper et al. (2012) and Kliem and Kriwoluzky (2014), among others. For monetary policy, we consider four alternative rules: a Taylor-type rule, strict inflation targeting, NGDP growth rate targeting (whereby the monetary authority targets the growth rate of NGDP), and NGDP level targeting (with the target being the level of NGDP). Our analyses are conducted both under the assumption that monetary policy can be adjusted so that the targets of NGDP and inflation are always achieved and under the alternative case where the economy may deviate from these targets in the short run (e.g., in downturns, the nominal GDP growth rate is lower than its target and/or the actual inflation rate is lower than the inflation target).

Our results can be summarized as follows. Under NGDP targeting, the fiscal policy rules become, de facto, a function of inflation rather than real GDP. Therefore, the cyclicality of tax rates is not clear and depends on the type of the underlying shock: a positive productivity shock reduces inflation and increases output while a negative demand shock reduces inflation and output. Under NGDP targeting, the tax rates will behave similarly because inflation falls in both cases. However, the outcomes regarding real economic activity are the exact opposites. We show that a reduction in the inflation rate leads to higher tax rates, which following a negative demand shock suggests that the tax rates increase in downturns. This outcome goes against the intention of preventing tax rates from rising in recessionary periods, and it makes the declines in output bigger and/or more prolonged: the rise in tax rates following an adverse shock aggravates the downturn.

We also analyze the government spending multiplier, analytically and numerically, in a model with NGDP targeting and compare it to the multipliers under inflation targeting and a Taylor rule. Analytically, nominal GDP targeting yields smaller government spending multipliers than inflation targeting in normal times (i.e. when the nominal interest rate is free to adjust to spending shocks). Second, under certain (plausible) conditions that we discuss in this study, NGDP targeting yields smaller spending multipliers than a standard Taylor rule. Numerically, we find that the government spending multiplier under NGDP targeting is clearly lower than the multipliers under inflation targeting and a Taylor rule, particularly when the economy deviates from the targets in the short run.

We obtain similar results regarding the tax multipliers. Compared to a Taylor rule and inflation targeting, reducing the labor tax rate, capital tax rate or the profit tax rate under NGDP targeting has smaller stimulative effects on output, particularly if the economy deviates from the targets in the short run. In the latter case, the differences between the tax multipliers under inflation targeting and a Taylor rule, on the one hand, and NGDP targeting, on the other, persist for a long period after the initial shock. The smaller fiscal multipliers under NGDP targeting reflect the constraints that these rules put on the behavior of real GDP. If the rise in government spending (or the tax rate cut) may push the economy past the NGDP target, then, in an attempt to bring the economy back to the NGDP target, tightening monetary policy will prevent a potentially larger increase in real GDP.

Our work adds to the recent literature on nominal GDP targeting. Kim and Henderson (2005) design a model that features optimization and monopolistic competition in both
the product and labor markets and find that nominal-income-growth targeting dominates inflation targeting for plausible parameter values. Dennis (2011) considers the general case where inflation expectations are a mixture of backward-looking and forward-looking terms, and finds that NGDP growth targeting does not lead to instability. Sumner (2014) argues that NGDP targeting provides the appropriate amount of liquidity to the economy and that it better addresses the dual mandate of the Federal Reserve. Garin et al. (2016) study the desirability of NGDP targeting using a New Keynesian model with price and wage rigidities. It is found that, following supply-side shocks and when wages are sticky relative to prices, NGDP targeting outperforms inflation targeting and a Taylor rule. Generally speaking, NGDP targeting produces small welfare losses and comes close to fully implementing the flexible price and wage allocation. Output gap targeting, however, tends to at least weakly outperforms NGDP targeting, but the differences in welfare losses associated with the two rules are small and there are instances when NGDP targeting is preferred.

Compared to a standard Taylor-type rule or a policy that responds strongly to inflation, Chen (2017) shows that by adopting nominal GDP growth rate targeting, not only the growth rate of real GDP is stabilized but so is the inflation rate. Billi (2017) compares NGDP level targeting to strict price level targeting in a small New Keynesian model, with the central bank operating under optimal discretion and facing a zero lower bound (ZLB) constraint. It is shown that, if the economy is only subject to temporary inflation shocks, then NGDP level targeting may be preferable. However, following persistent supply and demand shocks, strict price level targeting may be preferable. Furthermore, during ZLB episodes, nominal GDP level targeting leads to larger falls in nominal GDP.

Frankel (2014) suggests that NGDP targeting may be more appropriate for middle-sized middle-income countries as opposed to large advanced economies. The main idea behind this proposal is that middle-sized middle-income countries often face supply-side shocks and terms of trade shocks that call for abandoning inflation targets and exchange rate regimes. Sheedy (2014) argues that nominal GDP targeting improves the functioning of financial markets when incomplete contracts are written in terms of money: as households’ nominal income levels are insulated from aggregate real shocks, this policy essentially allows for risk sharing by stabilizing the debt-income ratio. By assuming that the central bank has imperfect information about the output gap, Beckworth and Hendrickson (2016) modify an otherwise standard New Keynesian model and show that nominal GDP targeting generates less volatility in both inflation and the output gap compared to a Taylor rule.

In light of this disagreement in the literature about the desirability of nominal GDP targeting, we provide analysis using a quantitative model (and for the government spending multiplier, analytical analysis as well) where fiscal policy is in the heart of the discussion. The aim of this analysis is not only to fill an important gap in the NGDP targeting literature but also to shed light on the applicability of this regime from the perspective of fiscal policy and the interactions with monetary policy management. And, by so doing, to help in investigating the desirability of nominal GDP in comparison with other monetary policy rules.

Our analysis of the government spending multiplier contributes to the recent literature
on this matter. Christiano et al. (2011) show that the multiplier is slightly above one if the nominal interest rate follows a standard interest-rate rule, but becomes considerably larger than one if the nominal interest rate does not respond to an increase in government spending, as may happen at the ZLB. The effects of the duration of interest rate peg on the size of the spending multiplier within a New Keynesian framework have been addressed by Carlstrom et al. (2014). They compare the results when the monetary-fiscal expansion lasts a certain number of periods (“deterministic duration”) with the results when the expansion is stochastic (with a mean that is equal to the fixed number of periods under the deterministic setup) and find the size of the stochastic multiplier to be large and clearly larger than the deterministic multiplier.

Zubairy (2014) studies the government spending multiplier in a stochastic general equilibrium model that features distortionary taxes and finds that the impact government spending multiplier is 1.07. In addition, private consumption responds positively to government spending shocks and the impact multipliers for labor and capital taxes are 0.13 and 0.34, respectively. At longer horizons (3-5 years), however, tax rate cuts become more stimulative than government spending. Bouakez et al. (2017) find a multiplier of 2.3 in a model with public investment and suggest that ignoring the investment component of the 2009 Stimulus Package leads to underestimation of the multiplier by nearly a half.

Most recently, Leeper et al. (2017) study the fiscal multipliers in four nested models: a simple real business cycle model, a standard new Keynesian model, a new Keynesian model with non-savers and a new Keynesian model with government spending in the utility function. They use Bayesian analysis and U.S. data in a DSGE model that includes fiscal details and two monetary-fiscal policy regimes. The authors find similar short-run output multipliers across regimes but significantly smaller multipliers after 10 years under the active monetary/passive fiscal regime than under the passive monetary/active fiscal regime; the average present values of the multipliers are 0.7 and 1.9, respectively.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy. Section 3 discusses fiscal and monetary policy rules, including NGDP targeting. Section 4 presents analytical results regarding the government spending multiplier. Section 5 presents the calibration of the model. Section 6 outlines the numerical results. Impulse-response functions are also shown in this section. Section 7 concludes.

2. The Model

The economy is populated by a continuum of infinitely-lived households, intermediate-good firms, final-good firms, a government and a monetary authority. Intermediate-good firms are monopolistically-competitive that produce differentiated products and sell them to final-good firms. These firms also face adjustment costs of prices. Final-good firms operate in a perfectly-competitive environment; they transform intermediate goods into final goods using a constant return to scale technology. In addition, throughout this paper, we let \( \bar{x} \) be the deterministic steady-state value of any variable \( x_t \).
2.1. Households

In each period \( t \), the representative household derives utility from consumption \( c_t \), supplies labor \( n_t \), holds bonds \( b_t \) and capital \( k_t \). Then, the problem of the representative household is:

\[
\max_{\{b_t, c_t, k_t, n_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \eta_t u(c_t, n_t)
\]

with \( \beta (< 1) \) being the household’s subjective discount factor, \( \eta_t \) is a taste shifter that captures changes in the patience rate, \( u(c_t, n_t) \) is the period utility function and \( \mathbb{E}_t \) is the expectations operator. Maximization is subject to the sequence of budget constraints (in real terms):

\[
(1 + \tau^n_t) c_t + k_{t+1} + b_t = \left(1 - \tau^n_t\right) w_t n_t + \frac{R_{t-1} b_{t-1}}{\pi_t} + \left[1 + r_t - \delta - \tau_k^b (r_t - \delta)\right] k_t + (1 - \tau^n_t) \Pi_t
\]

where \( w_t \) is the real wage, \( R_t \) is the gross nominal interest rate on bonds, \( \pi_t \) is the gross inflation rate, \( r_t \) is rental rate of capital, \( \delta \) denotes the depreciation rate of capital, \( \tau^n_t, \tau^c_t \) and \( \tau^k_t \) stand, respectively, for the labor-income tax rate, consumption tax rate and capital-income tax rate. \( \Pi_t \) are the profits from owning the intermediate-good firms and \( \tau^b_t \) is the tax rate on these profits.

The optimal choices of consumption, labor, capital and bond holdings yield the following optimization conditions:

\[
- \frac{u_{n,t}}{u_{c,t}} = \left(1 \frac{1 - \tau^n_t}{1 + \tau^c_t}\right) w_t
\]

\[
u_{c,t} = \beta R_t \mathbb{E}_t \left( \frac{\eta_{t+1} u_{c,t+1}}{\eta_t} \frac{1 + \tau^c_t}{\pi_{t+1} + 1 + \tau^c_{t+1}} \right)
\]

\[
u_{c,t} = \beta \mathbb{E}_t \left( \frac{u_{c,t+1}}{\eta_t} \frac{\eta_{t+1}}{1 + \tau^c_t} \frac{1 + \tau^c_{t+1}}{\pi_{t+1} + 1 + \tau^c_{t+1}} \left[1 - \delta + r_{t+1} - \tau^k_{t+1}(r_{t+1} - \delta)\right] \right)
\]

where \( u_{c,t} \) is the marginal utility of consumption and \( u_{n,t} \) is the marginal utility of supplying labor in period \( t \). Equation (3) is the standard labor supply condition stating that, at the optimum, the marginal rate of substitution between labor and consumption equals the after-tax real wage. Equation (4) is the standard consumption Euler equation and condition (5) is the standard capital supply condition with capital-income taxation.

2.2. The Production Sector

As is standard in the literature, two types of firms operate in this sector: monopolistically-competitive intermediate-good firms who produce differentiated products and perfectly-competitive final-good firms who transform intermediate goods into final goods using a constant return to scale technology.

2.2.1. Final-Good Firms

Firms in this sector purchase a continuum of intermediate goods from intermediate-good producers, indexed by \( j \in (0, 1) \), and assemble them into final goods using the following technology:
\[ y_t = \left( \int_0^1 y_{j,t}^{\varepsilon} \, dj \right)^{\frac{1}{\varepsilon}} \]  

(6)

with \( y_{j,t} \) being the quantity of intermediate-good \( j \) that is purchased by a final-good firm and \( \varepsilon > 1 \) is the elasticity of substitution between two differentiated types of intermediate goods. Profit maximization by intermediate-good producers gives the following downward-sloping demand function for the product variety \( j \):

\[ y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\varepsilon} y_t \]  

(7)

where \( P_t = \left( \int_0^1 P_{j,t}^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \) is the Dixit-Stiglitz aggregate price level that results from cost minimization.

### 2.2.2. Intermediate-Good Firms

Each firm \( j \) hires labor and rents capital from households to produce a differentiated product using the following technology:

\[ y_{j,t} = z_t f(k_{j,t}, n_{j,t}) \]  

(8)

where \( y_{j,t} \) and \( z_t \) are output and total factor productivity (which is common to all firms), respectively. Each firm chooses its own price \( P_{j,t} \), labor \( n_{j,t} \) and capital \( k_{j,t} \) to maximize profits subject to the downward-sloping demand for its product. The pricing of an intermediate-good firm is subject to a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final good:

\[ \Phi_{j,t} = \frac{\varphi}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \pi \right)^2 y_t \]  

(9)

where \( \varphi \) is a parameter that governs the degree of price rigidity and \( \pi \) is the inflation rate in the deterministic steady state (see Krause and Lubik, 2007 for a similar approach).

The problem of the intermediate-good firm is given by:

\[
\max_{\{k_{j,t}, n_{j,t}, P_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \eta_t \left[ \frac{\lambda_t}{\lambda_0} \left( \frac{P_{j,t}}{P_t} y_{j,t} - w_t n_{j,t} - r_t k_{j,t} - \frac{\varphi}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 y_t \right) \right]
\]

(10)

with \( \lambda_t = \frac{\eta_{t+1}}{(1+\gamma)} \) and \( \beta_t = \frac{\gamma_t}{\eta} \frac{\lambda_{t+1}}{\lambda_t} \) being the stochastic discount factor between two consecutive periods. Maximization is subject to the demand curve for the firm’s product (7) and the production technology (8). Profit maximization gives the following demand conditions for labor and capital, respectively:

\[ z_t f_{n_{j,t}} m c_{j,t} = w_t \]  

(11)

\[ z_t f_{k_{j,t}} m c_{j,t} = r_t \]  

(12)

where \( m c_{j,t} \) is the real marginal cost of firm \( j \). As expected, the firm hires labor and rents capital so that the marginal product of each input is a markup over its factor price.

In a symmetric equilibrium, in which all firms set the same price, Rotemberg pricing leads to the following forward-looking Phillips curve:
\[ 1 - \varphi (\pi_t - \bar{\pi}) \pi_t + \beta \varphi \mathbb{E}_t \left[ \frac{\eta_{t+1}}{\eta_t} \left( \frac{u_{c,t+1} (1 + \tau_t^c)}{u_{c,t} (1 + \tau_t^{c+1})} \right) \left( \pi_{t+1} - \bar{\pi} \right) \frac{y_{t+1}}{y_t} \right] = \varepsilon \left( 1 - mc_t \right) \] (13)

which differs from the standard forward-looking Phillips curve due to consumption taxation and the taste shifter. With fully flexible prices (\( \varphi = 0 \)) or when \( \pi_t = \bar{\pi} \) for all \( t \), equation (13) collapses to the familiar condition, \( mc_t = \frac{\varepsilon - 1}{\varepsilon} \). Therefore, in the absence of price adjustment costs, the real marginal cost equals the inverse of the steady-state price markup.

Finally, under the assumption of a Cobb-Douglas production function \( y_{j,t} = z_t k_{j,t}^\alpha n_{j,t}^{1-\alpha} \), which we will maintain in this study, the real marginal cost can be written as:

\[ mc_{j,t} = \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \frac{r_t^\alpha w_t^{1-\alpha}}{z_t} \] (14)

which reflects the cost of labor and capital proportionally. Without capital, it becomes \( mc_{j,t} = \frac{w_t}{z_t} \) as is standard in the New Keynesian model with linear-in-labor technology. In addition, since the real wage, the rental rate of capital and productivity are common to all firms, the marginal cost is the same for all firms (namely, \( mc_{j,t} = mc_t \) for all \( j \)).

2.3. Market Clearing

In equilibrium, the resource constraint of the economy reads:

\[ z_t f(k_t, n_t) + (1 - \delta) k_t = c_t + k_{t+1} + g_t + \frac{\varphi}{2} (\pi_t - \bar{\pi})^2 z_t f(k_t, n_t) \]  (15)

Government expenditures are financed via taxes and borrowing. Therefore, the government budget constraints is given by:

\[ g_t + \frac{R_{t-1} b_{t-1}}{\pi_t} = \tau_t w_t n_t + \tau_t^c c_t + \tau_t^k (r_t - \delta) k_t + \tau_t^\Pi \Pi_t + b_t \]  (16)

with government spending evolving according to an exogenous process that will be specified in what follows.

3. Monetary and Fiscal Policy Rules

This section outlines the rules that interchangeably describe monetary policy and the fiscal policy rules that govern the behavior of tax rates in the economy. We then discuss the implications of NGDP targeting for the shape of these fiscal policy rules. This section is concluded by defining the private-sector equilibrium.

3.1. Monetary Policy Rules with Full Adjustment

We start by describing a standard Taylor-type rule (TR), and then move to strict inflation targeting (IT), nominal GDP growth rate targeting (NGDPG) and nominal GDP level targeting (NGDPL). By so doing, we assess NGDP targeting in comparison with other standard
monetary policy rules. For the interest-rate rule, we assume that the nominal interest rate responds to deviations of inflation and output from their steady-state values as follows:

\[
\ln \left( \frac{R_t}{R_T} \right) = \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y_T} \right),
\]

(17)

with \( \phi_\pi \) and \( \phi_y \) being the coefficients of the inflation rate and output, respectively.

Under strict inflation targeting, the monetary authority commits to a specific inflation target \( (\pi^T) \) at all dates and states:

\[
\pi_t = \pi^T
\]

(18)

and the nominal interest rate is then adjusted to achieve the target.

Nominal GDP growth rate targeting assumes that the monetary authority commits to a certain growth rate of nominal GDP. Letting \( Y_t \) being nominal output, this rule reads:

\[
\frac{Y_t}{Y_{t-1}} = g_Y^T
\]

(19)

or, using the definition of nominal output \( (Y_t = P_t y_t) \), we have:

\[
\frac{y_t}{y_{t-1}} \pi_t = g_Y^T
\]

(20)

with \( g_Y^T \) being the gross growth rate target of nominal GDP.

Finally, under nominal GDP level targeting, nominal GDP \( (Y_t) \) is set as follows:

\[
Y_t = Y^T
\]

(21)

with \( Y^T \) being the target of nominal output. This condition can also be rewritten as:

\[
\frac{y_t}{y_{t-1}} \pi_t = 1
\]

(22)

One implication of condition (22) is that if prices are fully rigid or the planner attempts to pursue a zero inflation rate policy (i.e. \( \pi_t = 1 \)), then output will be fixed. For this reason, an expansionary fiscal policy will not affect output.

According to condition (20), in the deterministic steady state, we have \( \pi = g_Y^T \). Furthermore, \( \pi = \pi^T \) under inflation targeting. Therefore, while comparing NGDP growth rate targeting and inflation targeting, we will set \( \pi^T = g_Y^T \). This implies that the steady states under both rules are the same, which allows better comparisons between the various rules (namely, any differences between the rules will not result from different steady state values). Needless to say, this setting only makes the steady-state outcomes the same across rules, but it does not force these rules to behave similarly outside the steady state.
3.2. Monetary Policy Rules with No Full Adjustment

So far, the analyses assume that the economy always operates at the targets of the nominal GDP growth rate, nominal GDP level or inflation rate. For that to occur, monetary policy should be manipulated in a way that ensures that each target is met. Furthermore, for the actual growth rate of nominal GDP to be at the target at all times, it must be the case that in the face of a shock that leads to a decline in output, inflation rises so that condition (20) holds. While this scenario is likely following adverse supply-side shocks, it is less likely following adverse demand shocks; one would expect a drop in demand to reduce both real output and inflation, which calls for violating conditions (20) and (22). If monetary policy can be fully and instantaneously adjusted to accommodate the decline in real output by adopting a policy that leads to a rise in inflation (e.g. easing monetary policy), then the target can be achieved. However, since the effects of monetary policy are likely to occur with a lag, it is more conceivable to assume that there will be simultaneous declines in inflation and the growth rate of real GDP. By extension, there will be a decline in the growth rate of nominal GDP beyond its target, at least in the short term.

Indeed, Figure A.1 depicts 1) the difference between the (quarterly) growth rate of nominal GDP and the average growth rate of nominal GDP over the period 1960:1-2015:4, which we use as the first measure for the nominal GDP growth rate target, and 2) the difference between the (quarterly) growth rate of nominal GDP and the 4-quarter moving average of the growth rate of nominal GDP over the same sample period, which is the alternative measure of the nominal GDP growth rate target. As expected, the figure illustrates clear fluctuations in the gap between the actual and the “target” growth rate of nominal GDP. In particular, this gap tends to be negative in recessions and positive otherwise.

Given these observations, this subsection modifies the monetary policy rules in a way that allows for the actual growth rate of NGDP, NGDP level, inflation rate to deviate from their targets in the short run, but with adjustment back to the targets in the long run. In addition, it is assumed that each target is achieved in the deterministic steady state of the model. Therefore, the model that we describe in subsection 3.1 (where we assume full adjustment of monetary policy to allow the targets to hold) and the model without full adjustment will have the same steady state values. In other words, the degree of adjustment matters only following shocks. This modification allows for the possibility that, as a result of negative demand shocks, both real output and inflation decline in the short run, which is the most likely outcome.

This modification does not alter the interest-rate rule, and therefore it continues to be given by condition (17). However, under strict inflation targeting we now have:

\[ \pi_t \theta_t = \pi^T \]

where \( \theta_t \) measures the deviation between the actual inflation rate and its target. With \( \theta_t > 1 \), the actual inflation rate falls short of the target and vice versa. In the steady state we have \( \bar{\theta} = 1 \), and therefore the economy operates at the target.
With nominal GDP growth rate targeting:
\[
\frac{Y_t}{Y_{t-1}} \omega_t = g_Y^T \tag{24}
\]
or, using the definition of nominal output \((Y_t = P_t y_t)\):
\[
\frac{y_t}{y_{t-1}} \pi_t \omega_t = g_Y^T \tag{25}
\]
with \(\omega_t\) serving as the adjustment variable for the growth rate target of NGDP. For \(\omega_t > 1\), the actual growth rate of NGDP is lower than its target. The opposite holds with \(\omega_t < 1\), and at the steady state we have \(\omega = 1\).

With nominal GDP level targeting, we have:
\[
Y_t \gamma_t = Y^T \tag{26}
\]
which can also be rewritten as:
\[
\frac{y_t}{y_{t-1}} \pi_t \gamma_t = 1 \tag{27}
\]
with \(\gamma_t\) being the adjustment variable. As above, \(\gamma = 1\) in the steady state and \(\gamma_t > 1\) when NGDP is below its target (and vice versa).

As will be demonstrated in what follows, the size of \(\omega_t, \omega_t, \gamma_t\) is crucial for the behavior of tax rates, output and the effectiveness of fiscal policy in stimulating economic activity. This modification, however, does not change the deterministic steady state of the model relative to the model with full adjustment. For this reason, any differences between the results that are obtained under both scenarios will not stem from differences in the deterministic steady states but rather from the dynamics of the model following shocks.

We close this subsection with a note on NGDP growth rate targeting under full price rigidity. At the steady state, condition (25) becomes \(\pi = g_Y^T\). Therefore, if the growth rate target of nominal GDP is positive (i.e. \(g_Y^T > 1\)), the inflation rate will be positive as well. For this reason, under NGDP growth rate targeting, fully rigid prices (namely, a zero inflation rate) will not be consistent with this condition, and thus it will be ruled out in what follows. On the other hand, this restriction does not apply to NGDP level targeting.\(^1\)

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\(^1\)Consistent with standard business cycle models, our framework does not include a trend growth. If we allow for a trend growth \(A(>1)\), then we have \(Y_t = Y^T A^t\), implying that the target for nominal GDP grows over time. Using the definition of nominal GDP, this condition can then be rewritten as \(w_{y_{t-1}} \pi_t = A\), and it differs from (22) because of the right-hand side of this equation. With a trend growth, NGDP level targeting becomes similar to NGDP growth rate targeting (in fact, the two rules coincide if \(A = g_Y^T\)). With a trend growth, fully sticky prices \((\pi_t = 1)\) is not consistent with the NGDP level rule, a reminiscent of the issue that we observe with NGDP growth rate targeting. If instead we allow for a growth in productivity \((y_t = Z_t A^t f(k_t, n_t))\), then NGDP growth rate targeting and NGDP level targeting, respectively, give \(w_{y_{t-1}} \pi_t = g_Y^T\) and \(w_{y_{t-1}} \pi_t = 1\), which are the same as in the model without trend.
3.3. Fiscal Policy Rules

The tax rates are assumed to respond to deviations of output and the public debt from their steady-state values, as follows:

\[ \ln \left( \frac{\tau_n^t}{\tau_n^{t-1}} \right) = \rho_n \ln \left( \frac{\tau_n^{t-1}}{\tau_n^{t-2}} \right) + (1 - \rho_n) \lambda_n \ln \left( \frac{y_t}{y_{t-1}} \right) + (1 - \rho_n) \delta_n \ln \left( \frac{b_{t-1}}{b_t} \right) \]  
(28)

\[ \ln \left( \frac{\tau_k^t}{\tau_k^{t-1}} \right) = \rho_k \ln \left( \frac{\tau_k^{t-1}}{\tau_k^{t-2}} \right) + (1 - \rho_k) \lambda_k \ln \left( \frac{y_t}{y_{t-1}} \right) + (1 - \rho_k) \delta_k \ln \left( \frac{b_{t-1}}{b_t} \right) \]  
(29)

\[ \ln \left( \frac{\tau_p^t}{\tau_p^{t-1}} \right) = \rho_p \ln \left( \frac{\tau_p^{t-1}}{\tau_p^{t-2}} \right) + (1 - \rho_p) \lambda_p \ln \left( \frac{y_t}{y_{t-1}} \right) + (1 - \rho_p) \delta_p \ln \left( \frac{b_{t-1}}{b_t} \right) \]  
(30)

Fiscal policy rules along these lines have been extensively used in the literature (e.g. Leeper et al., 2010, Leeper et al., 2012 and Kliem and Kriwoluzky, 2014). The response to the deviation of output from the steady state captures the effects of the state of the economy on the tax rate, and the response to the debt captures the desire to stabilize the debt.

For the consumption tax rate, we follow Leeper et al. (2010) and Leeper et al. (2012) by letting the tax rate on consumption be an exogenous process. As explained in these studies, U.S. federal consumption taxes do not appear to respond to changes in other variables (such as output and the government debt). They are mostly excise taxes on specific products and are mainly used for special funds. Thus, they do not adjust to changes in current output or government debt. Furthermore, they are small in magnitude relative to other tax rates (for example, in Leeper et al. (2010), the consumption tax rate is less than 3%). We thus let the consumption tax rate be given by:

\[ \ln \left( \frac{\tau_c^t}{\tau_c^{t-1}} \right) = \rho_c \ln \left( \frac{\tau_c^{t-1}}{\tau_c^{t-2}} \right) + \varepsilon_{\tau_c,t} \]  
(31)

with \( \varepsilon_{\tau_c,t} \) being the shock to the consumption tax rate and \( \varepsilon_{\tau_c,t} \sim N(0, \sigma_c^2) \).

With NGDP growth rate targeting, we have:

\[ \frac{y_t}{y_{t-1}} = \frac{g_T y_t}{\bar{y}} \pi_t \omega_t \]  
(32)

and, therefore, the deviation of current real output from its steady-state level can be written as a function of the previous-period real output, current inflation, the steady-state value of real output, the growth rate target of nominal output and the adjustment variable. With this characterization, each tax rate rule now takes the form:

\[ \ln \left( \frac{\tau_i^t}{\tau_i^{t-1}} \right) = \rho_i \ln \left( \frac{\tau_i^{t-1}}{\tau_i^{t-2}} \right) + (1 - \rho_i) \lambda_i \ln \left( \frac{g_T y_t}{\bar{y}} \right) + (1 - \rho_i) \delta_i \ln \left( \frac{b_{t-1}}{b_t} \right) \]  
(33)

and \( i = n, k, p \). In the full adjustment case (\( \omega_t = 1 \)), the inflation rate emerges as the only non predetermined variable in this condition, implying that the fiscal authority will essentially respond to the behavior of the inflation rate during period \( t \), which does not align with the
intention. Adding the adjustment variable $\omega_t$ allows for more realism and for inflation not to be the only time-$t$ determinant of the tax rates. The dependence of the tax rate on inflation under NGDP growth rate targeting is the first important result that we report in this paper, and it illustrates how NGDP targeting tightens the relationship between fiscal policy and monetary policy (through the performance of inflation).

The implications of having the inflation rate in the tax rate rules can be significant. For illustration, assume that $\lambda_i > 0$ (which, as will be reported in what follows, is indeed the case), and consider two types of shocks that reduce inflation in period $t$. First, a positive supply shock reduces inflation and raises real output above steady state. Since output rises, the traditional fiscal policy rule suggests a rise in the tax rate. Similarly, since inflation declines, the modified fiscal policy rule (33) also calls for increasing the tax rate. Therefore, both rules lead to the same conclusion and NGDP targeting does not affect the behavior of the tax rate, at least qualitatively.

Consider now a negative demand shock that reduces inflation and real output: the modified fiscal policy rule calls for raising the tax rate. However, the tax rate in this case will rise when real economic activity is depressed, which may aggravate the decline in real output. Therefore, under this scenario, NGDP targeting effectively forces the pursuing of a contractionary fiscal policy in bad economic times instead of “leaning against the wind” to stimulate economic activity.

An event that leads to an increase in inflation will have the opposite effect. A negative supply shock reduces output and raises inflation, leading to lower tax rates. A positive demand-side shock increases both output and inflation. In this case, the modified tax rate rules imply lower tax rates even though the intent was to raise the tax rates in an upturn. For this reason, while the goal is to raise tax rates in good times and reduce them in bad times (and higher tax rates in good times allow for more room for cutting them in bad times), NGDP targeting may push the fiscal authority in the opposite direction in the aftermath of demand shocks. Broadly speaking, instead of a procyclical behavior of the tax rates, NGDP targeting makes the tax rates, de facto, behave countercyclically.

Nominal GDP targeting, thus, is very restrictive and may be detrimental. It prescribes the opposite of what should be done following demand-side shocks. And, since most U.S. recessions are demand-driven, this policy prescription can lead to undesirable consequences. At the very least, on face value, it does not appear that NGDP targeting and standard fiscal policy rules that respond to the state of real economic activity are compatible or produce the desired outcomes. These observations will be numerically tested in Section 6.

We next show that these observations are robust to two alternative specifications. First, with NGDP level targeting, the tax rate rules read:

$$\ln \left( \frac{\tau^n_t}{\tau^n_t} \right) = \rho_i \ln \left( \frac{\tau^n_{t-1}}{\tau^n_t} \right) + (1 - \rho_i) \lambda_i \ln \left( \frac{1}{b} \frac{\gamma_{t-1} y_{t-1}}{\gamma_{t-1} \pi_t} \right) + (1 - \rho_i) \delta_i \ln \left( \frac{b_{t-1}}{b} \right)$$

and $i = n, k, p$. As before, the behavior of each tax rate depends on the response of inflation.
to shocks as well the adjustment variable $\gamma_t$. If the economy always lies at the target ($\gamma_t = 1$ for all $t$), then the inflation rate will be the only time-$t$ variable in the tax rate rules. For this reason, the analyses above can be carried over to this specification too.

Second, if we assume that the various tax rates respond to changes in output as opposed to deviations of output from the steady state (namely, the tax rates respond to $\frac{yt_{y_t-1}}{yt_1}$), then the modified tax rate rules with NGDP growth rate targeting and NGDP level targeting can, respectively, be written as:

\[
\ln\left(\frac{\tau^i_t}{\tau^i_1}\right) = \rho_i \ln\left(\frac{\tau^i_{t-1}}{\tau^i_1}\right) + (1 - \rho_i)\lambda_i \ln\left(\frac{y^T_t}{\pi_t\omega_t}\right) + (1 - \rho_i)\delta_i \ln\left(\frac{b_{t-1}}{b}\right) \tag{35}
\]

\[
\ln\left(\frac{\tau^i_t}{\tau^i_1}\right) = \rho_i \ln\left(\frac{\tau^i_{t-1}}{\tau^i_1}\right) + (1 - \rho_i)\lambda_i \ln\left(\frac{\gamma_{t-1}}{\gamma_1\pi_t}\right) + (1 - \rho_i)\delta_i \ln\left(\frac{b_{t-1}}{b}\right) \tag{36}
\]

and $i = n, k, p$. In fact, under this specification, our observations are even clearer. With NGDP targeting, instead of the tax rates being a function of real economic activity, they become a function of the inflation rate. The adjustment variables $\omega$ and $\gamma_t$ are important too for the behavior of tax rates. If $\omega_t = \gamma_t = 1$ for all $t$, then each tax rate will be a function of current inflation only. The introduction of $\omega_t$ and $\gamma_t$ makes the tax rates also dependent on the deviations of the NGDP growth rate and NGDP level from their targets.\(^2\)

### 3.4. Adjustment Variables

We assume that $\omega_t, \gamma_t, \theta_t$ are functions of the state of the economy (represented by real output) as follows:

\[
\ln\left(\frac{s_t}{s}\right) = \rho_s \ln\left(\frac{s_{t-1}}{s}\right) + (1 - \rho_s)\mu_s \ln\left(\frac{yt}{y}\right) \tag{37}
\]

with $s = \omega, \gamma, \theta$. The parameters $\mu_\omega, \mu_\gamma, \mu_\theta$ govern the response of each adjustment variable to real output, and they will be set based on the empirical evidence in the calibration section. Starting from the non-stochastic steady state, if $\mu_s = 0$ or $\rho_s = 1$, then the corresponding variable remains at its non-stochastic steady-state value (which is 1). This corresponds to the “full adjustment” case in this paper. Following a negative shock that pushes real output below its steady state value, $\omega_t, \gamma_t, \theta_t$ are expected to rise above their steady-state values; therefore, the parameters $\mu_\omega, \mu_\gamma, \mu_\theta$ are expected to be negative.

---

\(^2\)The dependence of tax rates on inflation under NGDP targeting raises an important conceptual matter. With a given level of government spending and since debt is predetermined, the government budget constraint will be satisfied if either tax rates or real debt are adjusted. For the real debt to adjust, the price level should also adjust, which echoes the observations of Leeper (1991) and Sims (1994), among others, in the so-called Fiscal Theory of the Price Level (FTPL). Under NGDP targeting, tax rates will adjust if inflation responds to shocks. Therefore, under this regime, price adjustments are even more important than what FTPL typically predicts. Interestingly, NGDP targeting has been raised by “market monetarists”, and yet it provides support for a theory that was challenged by monetarist economists. Our model does not include money and thus formal analysis of this matter are not feasible, but one might view our findings as being in line with FTPL. Furthermore, with NGDP targeting, there are more interactions between fiscal policy and monetary policy, which also aligns with the FTPL.
3.5. The Private-Sector Equilibrium

Definition 1 (Equilibrium). Given the exogenous processes of \{z_t, g_t, \eta_t, \tau^c_t\} and a monetary policy rule, the private-sector equilibrium is a sequence of allocations \{b_t, c_t, k_t, mc_t, n_t, r_t, w_t, \pi_t, \tau^n_t, \tau^k_t, \tau^n_t\} that satisfy the equilibrium conditions (3)-(5), (11)-(13), (15)-(16) and (28)-(30).

In the case of non-full adjustment, this definition is expanded to include the variables \(\omega_t, \gamma_t, \theta_t\) and the three equations that govern their behavior (given by (37)). This concludes the description of the model.

4. Analytical Analyses- The Government Spending Multiplier

In this section, we provide analytical analyses about the effectiveness of fiscal policy with NGDP targeting. To do so, we derive the spending multiplier under NGDP targeting and compare it to the spending multiplier that is implied by inflation targeting and a Taylor rule. Following the literature (e.g. Christiano et al., 2011, Carlstrom et al., 2014), the model is solved using the Method of Undetermined Coefficients. To fix ideas, we consider a simplified version of the model without capital and without distortionary taxation. These assumptions allow us to compare our results to the benchmark findings of Christiano et al. (2011), Carlstrom et al. (2014) and Dupor and Li (2015), among others. We also follow these studies by log-linearizing the model around the deterministic steady state.

The general definition of the multiplier is then given by:

\[
\frac{dy_t}{dg_t} = 1 + \frac{1 - g \bar{c}_t}{g \bar{g}_t}.
\]

with \(\bar{x}_t\) being the log-deviation of any variable \(x_t\) from its deterministic steady state \(\bar{x}\), and \(g = \frac{\bar{g}_t}{\bar{g}}\) denoting the steady-state government spending-output ratio. As expected, the size of the multiplier depends on the response of consumption to government spending disturbances. If consumption rises ("crowding in"), then the multiplier will be larger than one. If it falls ("crowding out"), the multiplier will be less than one. And, if consumption does not respond to government spending shocks, then the multiplier will be exactly one. In addition, when all output is absorbed by the government \((g = 1)\), the multiplier equals one.

Throughout the analysis, we make use of the following period utility function:

\[
u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi n_t^{1+\nu} \frac{1+\nu}{1+\nu}
\]

with \(\sigma\) being the consumption curvature parameter, \(\chi\) is a scaling parameter that measures the relative weight on the disutility of labor and \(\nu\) being the inverse of the intertemporal elasticity of labor supply.

While the Taylor-type rule does not constitute a monetary policy regime per se, it is useful to compare the results with NGDP targeting and inflation targeting to the results with a Taylor-type rule (the latter has been the workhorse of previous studies on the multiplier). As
shown in Appendix B.4, the cumulative government spending multipliers \( (M) \) for inflation targeting, Taylor rule and NGDP targeting are, respectively, given by:

\[
M^{IT} = \left( \frac{\sigma}{\sigma + \nu(1-g)} \right) \frac{1}{1-\beta\rho} \tag{40}
\]

\[
M^{TR} = \left( \frac{\kappa \sigma (\phi - \rho) + \sigma (1-\rho)(1-\beta\rho)}{\kappa (\phi - \rho) [\sigma + \nu(1-g)] + [\sigma (1-\rho) + \phi_y] (1-\beta\rho)} \right) \frac{1}{1-\beta\rho} \tag{41}
\]

\[
M^{NGDPG} = M^{NGDPL} = \left( \frac{\sigma}{\sigma + \nu(1-g)} \right) \frac{1-\lambda}{(1-\beta\rho)(1-\beta\rho\lambda)} \tag{42}
\]

with \( 0 < \lambda < 1 \), \( \kappa \) being the slope of the log-linearized Philips curve and \( \rho \) is the persistence parameter of government spending in the simplified model. For the Taylor rule, the standard determinacy condition \( (\phi > 1) \) is assumed. Since \( \frac{1-\lambda}{(1-\beta\rho\lambda)} < 1 \), the right-hand side of condition (42) is less than the right-hand side of condition (40). Therefore, NGDP targeting is associated with smaller spending multipliers. The ranking of the spending multiplier under a Taylor rule depends on the size of \( \phi_y \) vs. \( (1-\rho)\nu(1-g) \). In particular, if \( (1-\rho)\nu(1-g) > \phi_y \), then \( M^{TR} > M^{IT} > M^{NGDP} \). For example, this occurs if \( \phi_y = 0 \), as is typically assumed in the literature on the government spending multiplier.\(^3\)

We also note that these results are based on a simplified version of the model and that some results with the full model may differ. The analytical results are not only useful to fix ideas but also to allow for comparisons with recent studies on the government spending multiplier that have relied on similar frameworks and a solution methodology as the one that we use in this section of the paper. In what follows, we present the government spending multiplier in a medium-scale model that includes capital and distortionary taxation.

### 5. Calibration

A summary of the parameter values is presented in Table 1. The time unit is a quarter and the discount factor \( \beta \) is set so that the annual interest rate is roughly 4%. As in the analytical section, households’ preferences are governed by (39), where the disutility-of-labor parameter \( \chi \) is set so that the steady-state value of \( n \) is 0.21 (which corresponds to a workweek of 35 hours, the average number of weekly hours worked over the period 1964:1-2015:4). The parameter \( \nu \) is set so that the labor supply elasticity is 2, which helps in capturing the volatility of total hours in a model with no extensive margin, as is the case in this paper.

\(^3\)We briefly comment on the case of liquidity trap (constant nominal interest rate). With a constant nominal interest rate, the government spending multiplier is independent of the degree of adjustment and is the same for all monetary policy rules (see Appendix B.5). In itself, this finding weakens the appeal of nominal GDP targeting as a desirable policy regime in economic downturns. A caveat here is in order: this analysis assumes that on exit from the constant nominal interest rate state, all policy rules imply the same nominal interest rate, which may not be the case. If the probability of a constant nominal interest rate is very high, this result becomes more accurate.
Firms produce using the production technology \( y_t = z_t f(k_t, n_t) \), with:
\[
f(k_t, n_t) = k_t^\alpha n_t^{1-\alpha}
\]
and \( \alpha \) being the elasticity of output with respect to capital. As in Cooley and Prescott (1995), the deprecation rate of physical capital is set in order to match the investment-capital ratio in the National Income and Product Accounts (NIPA). For 1960-2015, the implied annual capital depreciation rate is 11%; therefore, \( \delta = 0.026 \).

We study impulse responses to shocks to government spending, the households’ discount factor and total factor productivity (TFP). Government spending, the households’ discount factor and TFP are, respectively, governed by the following AR(1) processes:
\[
\ln \left( \frac{g_t}{\bar{g}} \right) = \rho_g \ln \left( \frac{g_{t-1}}{\bar{g}} \right) + \varepsilon_{g,t} \tag{44}
\]
\[
\ln \left( \frac{\eta_t}{\bar{\eta}} \right) = \rho_\eta \ln \left( \frac{\eta_{t-1}}{\bar{\eta}} \right) + \varepsilon_{\eta,t} \tag{45}
\]
\[
\ln \left( \frac{z_t}{\bar{z}} \right) = \rho_z \ln \left( \frac{z_{t-1}}{\bar{z}} \right) + \varepsilon_{z,t} \tag{46}
\]
with \( \varepsilon_{g,t}, \varepsilon_{\eta,t} \) and \( \varepsilon_{z,t} \) being the shocks to government spending, the discount factor and total factor productivity, respectively. These shocks are \( \varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2) \), \( \varepsilon_{\eta,t} \sim \mathcal{N}(0, \sigma_\eta^2) \) and \( \varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2) \), and their standard deviations are set in each simulation to account for the volatility of the quarterly U.S. GDP (which is 0.015 for the investigated period). The values of the AR(1) coefficients of government spending (\( \rho_g \)) and the shock to TFP (\( \rho_z \)) are standard in the literature. To calculate \( \rho_\eta \), we first calculate the time-varying value of the discount factor \( \beta_\eta \) from the consumption Euler equation (4) and then find the implied AR(1) coefficient.

The deterministic steady-state value of government spending \( \bar{g} \) is set so that it is 20\% of deterministic steady-state GDP, which corresponds to the average government spending-GDP ratio over the period 1960-2015. The steady-state value of debt \( \bar{b} \) is obtained so that \( \bar{b}/\bar{y} \) is 39.2\%, which is the average of the gross federal debt that is held by the public as percentage of GDP for the same period. These figures are based on Table 18 of the 2016 Economic Report of the President.

Unless otherwise stated, the data are available at the FRED database of the Federal Reserve Bank of St. Louis. We set \( \tau_k = 0.30 \) and \( \tau_p = 0.34 \). The value of \( \tau_k \) is based on the calculations of McDaniel (2007) of annual data and, following Molodtsova et al. (2008), this series has been converted into quarterly data using quadratic interpolation. The value of \( \tau_p \) is set to match the average corporate tax rate from the Bureau of Economic Analysis (BEA) data. The labor-income tax rate data are obtained from Karabarbounis (2014), and we calculate the labor-income tax rate following the aforementioned study. The implied average labor-income tax rate is \( \tau_n = 0.20 \).

The coefficients of the rules of the adjustment variables and the coefficients of the tax rate rules are set based on the empirical evidence (see Tables A.1-A.2). In particular, in
order to allow for tax rate smoothing and the dependence of the tax rates on the state of the economy and the public debt, we regress the cyclical component of each tax rate on its lagged value, the output gap and the real debt. The implied parameter values are reported in the second panel of Table 1. The labor, capital and profit tax rates positively respond to output and debt deviations from their respective steady-state values. Also, $\phi_\pi$, $\phi_y$ are negative, in line with expectations. The latter is consistent with inflation and NGDP being below targets in downturns.

The values of the Taylor rule coefficients, $\phi_\pi$ and $\phi_y$, are standard in the literature. The value of the price rigidity parameter ($\varphi$) is consistent with a price duration of 2.5 quarters and the parameter $\varepsilon$ delivers a steady-state markup rate of 20%, both in line with previous studies. The setting of $g^T_Y$ and $\pi^T$ imply annual inflation rate and NGDP growth rate target rates of 5%. While the target of the inflation rate is higher than the average historical inflation rate in the U.S., this setting allows all models to have the same deterministic steady-state outcomes (which is consistent with the steady-state versions of conditions ((18) and (20)). Furthermore, we note that the main results of the paper hold if we assume an annual inflation rate target of 2%.

Finally, since the log-deviation of any variable $x_t$ from its steady state satisfies $\frac{x_t}{x} = 1 + \tilde{x}_t$, each fiscal policy rule can also be re-written as:

$$\ln \left(1 + \tilde{x}_t\right) = \rho_i \ln \left(1 + \tilde{x}_{t-1}\right) + \left(1 - \rho_i\right) \lambda_i \ln \left(1 + \tilde{y}_t\right) + \left(1 - \rho_i\right) \delta_i \ln \left(1 + \tilde{b}_{t-1}\right)$$

and $i = n, k, p$. We then deterend the data and use this formulation to calculate the coefficients of the tax rate rules that are reported in Table 1. For more details on the data sources and the analysis, see Appendix A.

Table 1: Values of the parameters- benchmark analyses. See Tables A.1-A.2. Note: with full adjustment: $\rho_\omega = \rho_\gamma = \rho_\theta = 1$ and $\mu_\omega = \mu_\gamma = \mu_\theta = 0$. 

<table>
<thead>
<tr>
<th>Preferences, Production and Government Spending</th>
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<td>$\beta$</td>
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<th>Tax Rate Rules</th>
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<table>
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<tr>
<th>Adjustment Conditions and Monetary Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_\omega$</td>
</tr>
<tr>
<td>0.31</td>
</tr>
</tbody>
</table>
6. Numerical Results

The main numerical results of the full model (Section 2 and Section 3) are presented here. We start by illustrating how, with nominal GDP targeting, adopting traditional tax-rate rules may lead to the opposite of the desired behavior of tax rates. We then present impulse responses following government spending shocks to assess the effectiveness of fiscal policy expansion under nominal GDP targeting versus other monetary policy rules. In addition, we present numerical evaluation of the tax multipliers under all monetary policy rules, and close by comparing the welfare implications of these fiscal policy rules.

6.1. Tax-Rate Rules With Nominal GDP Targeting

We present the behavior of tax rates to a negative government spending shock (that calls for a reduction in output and inflation), a negative TFP shock (that triggers a decline in output but a rise in inflation), and a rise in the households’ discount factor (which is expected to reduce output and inflation as households delay consumption). We report the results with full adjustment of monetary policy (under which the targets of nominal GDP are always achieved, namely $\omega_t = \gamma_t = 1$) and without full adjustment. For each scenario, the standard deviation of the shock is set to match the standard deviation of U.S. real GDP over the period 1960:1-2015:4.

Consider first a shock to government spending (Figure 1). On impact, output declines as a result of the reduction in government spending, with and without full adjustment. With full adjustment, a decline in output is associated with a rise in the inflation rate, which in turn leads to a reduction in the tax rates. In this respect, the tax rates behave as expected; nominal GDP targeting does not distort their paths, at least qualitatively. Furthermore, when the tax rates reach their lowest points, output slightly rebounds for few quarters before reverting to its steady state.

Under non-full adjustment, there is a fall in the inflation rate, which in itself triggers a rise in the tax rates (recall condition (33)). On the other hand, the decline in government spending pushes for lower debt and taxation, which in itself induces lower tax rates. On impact, the first effect dominates, partly because the coefficients of output in the tax rate rules ($\lambda_i$) are larger than the coefficients of the debt ($\delta_i$). As a result, the tax rates end up rising at the same time that output falls. Clearly, this behavior of the tax rates goes in the opposite direction of the intended path. Note that, in absolute values, the rise in the tax rates in the non-full adjustment case is smaller than the fall in these tax rates in the full adjustment case (for which the lower output and lower debt both induce a reduction in the tax rates). Furthermore, the fall in government spending in the non-full adjustment case prevents an even bigger rise in the tax rates.

When the discount factor rises, the results are even clearer (Figure 2). Real output falls under all scenarios, but the tax rates rise. For both types of shocks, the largest fall in output occurs with the largest increase in the tax rates (which occurs with NGDP level targeting and non-full adjustment). The results for the non-full adjustment case are consistent with
our previous analysis; they reflect the fall in the inflation rate following adverse demand shocks, thus generating higher tax rates under NGDP targeting. However, the results with full adjustment are different from what we observe following a reduction in government spending. This occurs because, with a negative discount factor shock, government spending remains unchanged and the government must resort to taxation and debt to finance the same level of government spending. Therefore, even if the rise in inflation with full adjustment calls for lower tax rates, the other effects (higher debt and the same level of government spending) in summation lead to higher tax rates.

Following a decline in TFP, the results are mixed. On the one hand, a decline in TFP implies a rise in inflation and a decline in real output, thus pushing for a decline in the tax rates. On the other hand, since government spending remains unchanged, either higher tax rates, higher debt or both are required to satisfy the government budget constraint. Under full adjustment, that leads to small changes in the tax rates. With non-full adjustment, however, considerably larger increases in the tax rates materialize (as the rise in inflation is muted). In addition, as debt is accumulated, the tax rates peak after nearly 8 quarters. We note that not the entire increases in the tax rates are attributed to NGDP targeting per se, but imposing this regime contributes to this behavior.

In sum, with nominal GDP targeting, tax rates tend to move in the opposite of the standard direction, which makes the declines in output bigger and/or more prolonged: the rise in tax rates following a recessionary shock only fuels the downturn. These numerical findings confirm the observations from subsection 3.3 and they highlight the potential problem of adopting standard fiscal-policy rules with NGDP targeting. In practice, with NGDP targeting, the tax rates do not necessarily respond pro-cyclically to real economic activity, but (mostly) move counter-cyclically. Therefore, this type of fiscal policy rules does not seem...
appropriate if nominal GDP targeting is adopted. Alternatively, one may view these results as indicating the inadequacy of NGDP targeting with standard tax rate rules.

6.2. Should Tax Rates Respond to Inflation?

A natural question to be asked is whether, with NGDP targeting, the tax rate rules should include an inflation component. To this end, consider the following amended tax rate rules with NGDP growth rate and NGDP level targeting, respectively:

\[
\ln \left( \frac{\tau_t^i}{\tau_{t-1}^i} \right) = \rho_i \ln \left( \frac{\tau_{t-1}^i}{\tau_t^i} \right) + (1 - \rho_i) \lambda_i \ln \left( \frac{g^T_t}{y^T_t} \frac{y_{t-1}}{\omega_i \pi_t} \right) + (1 - \rho_i) \delta_i \ln \left( \frac{b_{t-1}}{b_t} \right) + (1 - \rho_i) \psi_i \ln \left( \frac{\pi_t}{\pi} \right) \quad (48)
\]

\[
\ln \left( \frac{\tau_t^i}{\tau_{t-1}^i} \right) = \rho_i \ln \left( \frac{\tau_{t-1}^i}{\tau_t^i} \right) + (1 - \rho_i) \lambda_i \ln \left( \frac{1}{\tilde{y}_{t-1}} \frac{y_{t-1} \gamma_t}{\gamma_t \pi_t} \right) + (1 - \rho_i) \delta_i \ln \left( \frac{b_{t-1}}{b_t} \right) + (1 - \rho_i) \psi_i \ln \left( \frac{\pi_t}{\pi} \right) \quad (49)
\]

with \( \psi_i \) being the response of tax rates to inflation and \( i = n, k, p \).

In principle, adding the last term in each tax rate rule and appropriately setting the parameter \( \psi_i \) may prevent each tax rates from behaving counter-cyclically and allow the tax rates to move in the intended direction (notice that \( \pi_t \) enters each tax rate rule through the denominator of the second term and the numerator of the last term on the right-hand side, and thus it is possible to allow the latter term to offset the former). This modification, however, requires each \( \psi_i \) to depend on the exact scenario in hand: negative demand and supply shocks may not only require different values of \( \psi_i \), but potentially different signs as well. In this respect, the tax rate rules themselves become state dependent, which arguably weakens their appeal for guiding taxation policies. In addition, this setting requires knowledge of the source of the shock (government spending, TFP or a change in the patience rate), which
To test this possibility, we report the results with the inflation rate in the tax rate rules (see Figures A.2-A.4). For the sake of illustration, we search for the values of $\psi_i$’s under which tax rates do not rise at least in the first four quarters following the initial shock. While this choice is admittedly arbitrary, it is a good choice for conducting this experiment and it helps avoiding an unlikely immediate rise in the tax rates. We then compare these values of $\psi_i$ to the corresponding coefficients of real output ($\lambda_i$); see Table A.3.

As these figures show, all tax rates decline following negative shocks, which is consistent with the desired behavior of these tax rates. The latter, however, comes with a drawback: the values of $\psi_i$’s to bring about a pro-cyclical behavior by the tax rates change from one specification to the other, and appear artificial. For example: under NGDPG, the observed reduction in the tax rates under a negative government spending shock and non-full adjustment occurs with $\psi_n = 10\lambda_n, \psi_k = 12\lambda_k, \psi_p = 9\lambda_p$. On the other hand, for a TFP shock and non-full adjustment, we need $\psi_n = 38\lambda_n, \psi_k = 41\lambda_k, \psi_p = 45\lambda_p$. For a TFP shock and full adjustment, $\psi_n = -35\lambda_n, \psi_k = -42\lambda_k, \psi_p = -40\lambda_k$, etc. Not only that these are sizeable coefficients but they clearly change from one specification to the other, as expected. Furthermore, these values of $\psi_i$’s may not be unique as one could construct different combinations of these parameters to get a reduction in the tax rate following adverse shocks. For these reasons, it does not appear that artificially adding an inflation term in the tax rate rules will significantly improve their usefulness under NGDP targeting.\(^4\)

\(^4\) Similar arguments can be made if tax rates are designed to respond to nominal GDP instead of real GDP: in theory, a decline in nominal GDP may reflect a decline in real GDP with a declining/rising/constant price level, or a rise in real GDP that is coupled with a larger decline in the price level. It could also reflect...
6.3. Impulse Responses- Government Spending Shocks

To evaluate the effectiveness of fiscal policy under NGDP targeting, this section briefly discusses impulse response functions following a government spending shock (the time unit is a quarter). We then calculate the “impact government spending multiplier” that is implied by these impulse-response functions.

To allow for safe comparisons, we use the standard deviation of the shock to government spending with a Taylor rule that yields the standard deviation of U.S. GDP as the standard deviation of the shock for all monetary policy rules. In other words, we investigate how a shock of a particular size to government spending stimulate real output under the alternative monetary policy rules that are considered in this study.

As shown in Figure 4 and Figure 5, real output rises in response to a positive government spending shock, but total private absorptions (consumption and investment, denoted by \( c+i \)) decline on impact. The rise in output is matched by a rise in labor, on the one hand, and a decline in capital, on the other. These observations hold for all monetary policy rules and they confirm the traditional view that a rise in government spending (partially) crowds out the private sector.

The rules differ, however, in terms of the response of output to the rise in government spending. Under TR and IT and assuming full adjustment, the response of output is signifi-
Figure 5: The impulse responses to a positive one standard deviation government spending shock with non-full adjustment.

cantly larger on impact, but then all rules lead to the same impact after roughly 4 quarters. Without full adjustment, the responses of output under NGDP growth rate and NGDP level targeting are slightly weaker than under full adjustment and clearly weaker than the responses of output under IT and TR. In this case, the gaps persist for roughly two years, and thus NGDP targeting is dominated by other rules for a longer periods of time. All rules essentially imply no effects on real output after nearly 5 years.


The impulse response functions are useful as a basis for alternative measurement of the government spending multiplier. To this end, we use the impact spending multiplier, which is the rise in real output $h$ periods ahead ($\Delta y_{t+h}$) in response to a rise in government spending at the current period ($\Delta g_t$):

$$\text{Impact Spending Multiplier}_{t,t+h} = \frac{\Delta y_{t+h}}{\Delta g_t}$$ (50)

The impact government spending multiplier is presented in Figure 6 for all policy rules. With full adjustment, the NGDP targeting multipliers on impact are nearly half of the multipliers under IT and TR, but all multipliers align after nearly 4 quarters. Without full adjustment, the differences between the multipliers are bigger and they persist for nearly two years. This figure supports the findings from the impulse responses and it confirms that NGDP targeting does not yield bigger spending multipliers than the alternative policy rules. In fact, the opposite outcome is more likely.
The fact that the multipliers are mostly below one is as expected in this line of literature (particularly with separable preferences as the ones that we use in this study). The goal of this section is not to address the debate about the size of the government spending multiplier in quantitative models per se, but rather to compare the multipliers across monetary policy regimes (or rules) while holding other factors fixed. In addition, the results with full adjustment in this model (with capital and distortionary taxes) support the analytical analysis of Section 4: inflation targeting dominates NGDP targeting.

6.5. Tax Multipliers

In this subsection, we numerically evaluate the tax multipliers with all monetary policy rules. To this end, we specify a fully exogenous process for each tax rate as follows:

$$\ln \left( \frac{\tau_i^t}{\tau_i^t} \right) = \rho_i \ln \left( \frac{\tau_i^t-1}{\tau_i^t} \right) + \varepsilon_{\tau_i,t}$$  

with $i = n, c, k, p$. In each case, we change one tax rate only while holding the other tax rates (as well as government spending, TFP and the discount factor) at their steady-state values. As before, the standard deviations of the shocks are set so that the standard deviation of output equals the standard deviation of U.S. GDP when monetary policy follows a Taylor rule. We calculate the impact tax multipliers $h$ periods ahead using the following condition:

$$\text{Impact Tax Multiplier}_{t,t+h} = \frac{\Delta y_{t+h}}{\Delta T^i_t}$$  

with $T^i$ being the revenue from each tax instrument.
In so doing, we can compare the resultant tax multipliers with the government spending multiplier. Figure 7 presents the tax multipliers, and the main findings can be summarized as follows. First, regardless of the assumption about adjustment, the tax multipliers with NGDP targeting are lower than the corresponding multipliers with a Taylor rule and inflation targeting. Second, the gap between the TR and IT multipliers, on one hand, and NGDP targeting multipliers, on the other, are significantly larger in the non-full adjustment case. Furthermore, the gaps persist for an extended period of time. Third, the tax multipliers tend to be smaller than the government spending multiplier. The only exception is the consumption tax rate. Fourth, the tax rate on profits is associated with the smallest multiplier. This perhaps occurs because the profit tax rate only affects the government budget constraint but has no direct effect on the optimality conditions of either households or firms. These analyses, thus, confirm that NGDP targeting does not dominate other monetary policy rules from the point of view of fiscal policy; in fact, the opposite is true.

6.6. Welfare Analysis
To further evaluate the performance of NGDP targeting, we present some welfare analysis by comparing all monetary policy rules to the optimal policy solution. To this end, we first solve a Ramsey-type optimization problem and then evaluate the performance of each rule. We start by making a definition of the Ramsey problem.

Definition 2 (Optimal Policy Problem). Given the exogenous processes \( \{z_t, \eta_t, g_t\} \), the Ramsey planner chooses sequences of allocations \( \{b_t, c_t, k_t, mc_t, n_t, r_t, R_t, w_t, \pi_t, \tau_c, \tau_k, \tau_n\} \) to maximize (1) subject to (3)-(5), (11)-(13) and (15)-(16)

---

This reflects the fact that the consumption tax rate is relatively small and any given change in output translate into a bigger consumption tax multiplier.
Since the monopolistic power of firms allows for profits in equilibrium, and these profits do not affect the optimality conditions of agents at the margin, the government would want to tax these profits at the optimal rate of 100%. This policy allows for reducing some of the distortionary tax rates on households; see Guo and Lansing (1999) for a discussion. On this basis, our analysis in this section assumes that $\tau_t^p = 1.$

<table>
<thead>
<tr>
<th></th>
<th>TR</th>
<th>IT</th>
<th>NGDPL</th>
<th>NGDPL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Adjustment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td>1.60</td>
<td>1.61</td>
<td>1.85</td>
<td>1.87</td>
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<tr>
<td>TFP Shock</td>
<td>1.80</td>
<td>1.74</td>
<td>2.05</td>
<td>2.23</td>
</tr>
<tr>
<td>Discount Factor Shock</td>
<td>1.73</td>
<td>1.68</td>
<td>1.98</td>
<td>2.16</td>
</tr>
<tr>
<td><strong>Non-Full Adjustment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td>1.60</td>
<td>1.54</td>
<td>1.81</td>
<td>2.11</td>
</tr>
<tr>
<td>TFP Shock</td>
<td>1.80</td>
<td>1.74</td>
<td>2.01</td>
<td>2.30</td>
</tr>
<tr>
<td>Discount Factor Shock</td>
<td>1.73</td>
<td>1.67</td>
<td>1.94</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Table 2: Welfare analysis- the consumption equivalence (in percentage terms).
Note: the model with variable tax rates as given by conditions (28)-(31).

Table 2 presents the consumption equivalence (CE), which is the percentage by which consumption should be increased so that welfare under each policy rule is equal to welfare under the optimal policy. The results reveal the following. First, consistent with the previous analysis, both NGDP targeting rules are dominated by a Taylor rule and inflation targeting. Second, NGDP growth rate targeting outperforms NGDP level targeting, a reminiscent of our earlier results. Third, comparison between the cases with full adjustment and non-full adjustment does not allow for decisive conclusions about the welfare losses. For inflation targeting, the differences between full adjustment and non-full adjustment are relatively small, and for NGDP level targeting, the differences are the largest. Nominal GDP level targeting is clearly the worst policy rule, particularly when full adjustment is not possible. We conclude that the observation that nominal GDP targeting is not superior to a Taylor rule or inflation targeting is fully supported by the welfare analysis. To the contrary, nominal GDP targeting (particularly of the level of nominal GDP) induces larger welfare losses in this environment than other monetary policy rules.

7. Conclusions

This paper studies fiscal policy with nominal GDP (NGDP) targeting, which has been raised as a potential regime to adopt following the Great Recession. In particular, we study fiscal policy rules as well as the government spending multiplier and tax multipliers in a model with NGDP targeting and compare the results to those under inflation targeting, an interest rate rule and optimal monetary policy. To this end, we utilize a quantitative model with
nominal rigidities and taxes on labor income, capital income, profits and consumption. In addition, we consider two alternative cases; in the first case, the economy always operates at the targets of inflation and nominal GDP. In the second, short-term deviations from the targets are possible.

We show that nominal GDP targeting renders standard fiscal policy rules, whereby tax rates are supposed to respond to the state of the economy and the state of the public debt, a function of the inflation rate rather than real economic activity. One implication of this result is that, following adverse shocks that reduce output and inflation, standard tax rate rules will actually imply a rise as opposed to a decline in tax rates. The behavior of tax rates depends also on the degree to which monetary policy can be adjusted so that the economy remains at the nominal GDP target.

In addition, we illustrate that the government spending multiplier and the tax rate multipliers are larger under a Taylor rule and inflation targeting than under nominal GDP targeting, particularly if the targets of nominal GDP and inflation are not achieved in the short run. Furthermore, comparing all monetary policy rules to optimal policy leads to a similar conclusion: NGDP targeting (particularly of the level of NGDP) is dominated by a Taylor rule and inflation targeting.

The smaller multipliers under NGDP targeting reflect the constraints that nominal GDP targeting puts on the behavior of real GDP; following a rise in government spending, the desire to puts the economy on the track to achieve the nominal GDP target may prevent a potentially larger increase in real GDP. In addition, under NGDP targeting, the path of real GDP is closely tied to the path of inflation. In this respect, we demonstrate that if prices are fully sticky, then NGDP level targeting is associated with a zero spending multiplier. A Taylor rule, however, results in strictly positive multipliers when prices are fully sticky.

To our knowledge, this paper constitutes the first attempt to study the implications of nominal GDP targeting for fiscal policy in a quantitative model as the focus of previous work has been on monetary policy. We do not find any clear evidence in support of adopting this regime as far as fiscal policy is concerned. Since the adoption of any monetary policy regime should also take into account the implications for fiscal policy (as inflation targeting does), we conclude that NGDP targeting, while may have some advantages that are not in the core of this study, is not the right regime to follow from the perspective of fiscal policy. Also, our work does not indicate that NGDP targeting is the right regime to follow in economic downturns in an attempt to promote aggregate expenditures in the economy.

References


A. Data, Figures and Tables

A.1. Data

Unless otherwise indicated, the data are extracted from the FRED database of the Federal Reserve Bank of St. Louis. We use the following data series:

- Real GDP: Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate
- Nominal GDP: Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
- Debt: Federal Debt: Total Public Debt as Percent of Gross Domestic Product, Percent of GDP, Annual, Seasonally Adjusted
- Price Level: Gross Domestic Product: Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted
- Labor-Income Tax Rate: Authors’ calculations based on Karabarbounis (2014).
- AFCP: Corporate Profits After Tax (without IVA and CCAdj), Billions of Dollars, Annual, Seasonally Adjusted Annual Rate
- BFCP: Corporate profits before tax (without IVA and CCAdj), Billions of Dollars, Annual, Seasonally Adjusted Annual Rate
- Corporate Tax Rate: (BFCP-AFCP)/BCFP
A.2. Figures

Figure A.1: Left panel: the deviation between the actual growth rate of nominal GDP and the period average growth rate of nominal GDP. Right panel: the deviation between the actual growth rate of nominal GDP and a 4-quarter moving average of the growth rate of nominal GDP. Sample: 1960:1-2015:4. Shaded areas indicate NBER recession dates.

Figure A.2: Impulse-Responses to a negative government spending ($g_t$) shock. Note: the model with inflation ($\pi_t$) in the tax rate rules (conditions (48)-(49)).
Figure A.3: Impulse-Responses to a negative discount factor ($\eta_t$) shock. Note: the model with inflation ($\pi_t$) in the tax rate rules (conditions (48)-(49)).

Figure A.4: Impulse-Responses to a negative TFP ($z_t$) shock. Note: the model with inflation ($\pi_t$) in the tax rate rules (conditions (48)-(49)).
A.3. Tables

To calculate $\omega_t$, we use condition (24):

$$\omega_t = g^T_Y \frac{Y_{t-1}}{Y_t}$$

We approximate $g^T_Y$ using the average growth rate of nominal GDP over the period 1960:1-2015:4, and then use the series of nominal GDP to obtain the values of $\omega_t$ throughout the sample period. The values of $\gamma_t$ and $\theta_t$ are obtained in a similar fashion (using conditions (23) and (26), respectively). The results are summarized in Table A.1. We use ordinary least squares (OLS) estimation as the goal is to find correlations (and not necessarily causations) between $\omega_t, \gamma_t$ and $\theta_t$, on the one hand, and the state of the economy, on the other.

<table>
<thead>
<tr>
<th></th>
<th>NGDPG</th>
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<tbody>
<tr>
<td>$\rho_\omega$</td>
<td>0.3090***</td>
<td>0.5012***</td>
<td>0.1902***</td>
</tr>
<tr>
<td>$\mu_\omega$</td>
<td>-1.2062***</td>
<td>-0.2056***</td>
<td>-0.0347***</td>
</tr>
<tr>
<td>R-Sq.</td>
<td>0.7226</td>
<td>0.6470</td>
<td>0.1010</td>
</tr>
<tr>
<td>Obs.</td>
<td>223</td>
<td>223</td>
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</tbody>
</table>

Table A.1: Empirical results- OLS estimation of condition (37). Standard errors in parentheses. * Denotes significance at the 10% level. ** Denotes significance at the 5% level. *** Denotes significance at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>Labor Taxation</th>
<th>Corporate Taxation</th>
<th>Capital Taxation</th>
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<tbody>
<tr>
<td>$\rho_n$</td>
<td>0.8194***</td>
<td>0.6403***</td>
<td>0.8167***</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>0.6546***</td>
<td>0.6110***</td>
<td>0.6577***</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>0.1808*</td>
<td>0.2177**</td>
<td>0.3925***</td>
</tr>
<tr>
<td>J-Test (p-value)</td>
<td>0.4763</td>
<td>0.7839</td>
<td>0.5254</td>
</tr>
<tr>
<td>Obs.</td>
<td>188</td>
<td>188</td>
<td>188</td>
</tr>
</tbody>
</table>

Table A.2: Empirical results- Generalized Method of Moments (GMM) estimation of condition (47). Standard errors in parentheses. The list of instruments includes four lags of the cyclical components (HP filtered) of the corresponding tax rate, real GDP and real debt. * Denotes significance at the 10% level. ** Denotes significance at the 5% level. *** Denotes significance at the 1% level. The J-Test tests the hypothesis that the over-identifying restrictions are satisfied. The series for debt is available only since 1966:1.
<table>
<thead>
<tr>
<th></th>
<th>NGDPG</th>
<th>NGDPL</th>
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<tbody>
<tr>
<td><strong>Full Adjustment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>TFP Shock</td>
<td>-35, -42, -40</td>
<td>-5, -8, -7</td>
</tr>
<tr>
<td>Discount Factor Shock</td>
<td>-24, -35, -31</td>
<td>-8, -7, -13</td>
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<tr>
<td><strong>Non-Full Adjustment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td>10, 12, 9</td>
<td>3, 4, 2</td>
</tr>
<tr>
<td>TFP Shock</td>
<td>38, 41, 45</td>
<td>-13, -9, -19</td>
</tr>
<tr>
<td>Discount Factor Shock</td>
<td>25, 37, 42</td>
<td>-9, -11, -17</td>
</tr>
</tbody>
</table>

Table A.3: The size of inflation rate coefficient ($\psi_i$) relative to the size of real output ($\lambda_i$). Note: The model with the inflation rate in the tax rate rules as given by conditions (48)-(49). These coefficients are used to generate Figures A.2-A.4. Each triple represents the coefficients of the inflation rate in the rules of the labor tax rate, capital tax rate and profit tax rate, respectively (i.e. $\psi_n, \psi_k, \psi_p$, respectively).
B. Mathematical Appendix

This appendix outlines the simplified model that is used for deriving the spending multiplier analytically (Section 4). The model abstracts from capital and distortionary taxation. Also, since we study the impact of shocks to government spending, we let TFP and the discount factor be constant; therefore $z_t = 1$ and $\eta_t = 1 \forall t$. Furthermore, to simply the analytical analyses, we assume full adjustment and that prices are set in a staggered fashion as in Calvo (1983); in particular, a fraction $\varsigma$ of firms do not adjust their prices each period.

B.1. Households

The problem of households is to choose consumption $c_t$, bonds $b_t$ and labor $n_t$ to:

$$\max_{\{b_t, c_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \left( \frac{c_t^{1-\sigma}}{1-\sigma} - \chi n_t^{1+\nu} \right) \frac{n_t^{1+\nu}}{1 + \nu} \right)$$

subject to ($\Pi_t$ and $T_t$ are profits and net taxes, respectively):

$$c_t + b_t = w_t n_t + \frac{R_{t-1} b_{t-1}}{\pi_t} + \Pi_t + T_t$$

The first-order conditions yield the following optimality conditions:

$$c_t^{1-\sigma} = \beta R_t \mathbb{E}_t \left( \frac{c_{t+1}^{\sigma}}{\pi_{t+1}} \right)$$

$$\chi n_t^\sigma c_t = w_t$$

Log-linearization of conditions (B.3) and (B.4) around the deterministic steady state then gives:

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = \sigma (E_t \hat{c}_{t+1} - \hat{c}_t)$$

$$\nu \hat{n}_t + \sigma \hat{c}_t = \hat{w}_t$$

where $\hat{x}_t$ denotes the log deviations of each variable $x_t$ from its deterministic steady state.

B.2. Production Sector

Since the production function is linear, we have $\hat{y}_t = \hat{n}_t$. Then condition (B.6) becomes:

$$\nu \hat{y}_t + \sigma \hat{c}_t = \hat{w}_t$$

The labor demand condition (with a constant TFP) implies $\bar{m} \hat{c}_t = \hat{w}_t$. Therefore, we have:

$$\bar{m} \hat{c}_t = \nu \hat{y}_t + \sigma \hat{c}_t$$

The Phillips curve in log deviations is given by:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \bar{m} \hat{c}_t$$

Then, combining (B.8) and (B.9) gives:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa [\nu \hat{y}_t + \sigma \hat{c}_t]$$
which is the log-linearized version of the Phillips curve that we use in what follows. The parameter \( \kappa = (1 - \varsigma)(1 - \beta \varsigma)/\varsigma \) is the slope of this curve. Fully rigid prices imply \( \kappa = 0 \), and fully flexible prices imply \( \kappa \to \infty \).

**B.3. Market Clearing**

The economy-wide resource constraint, in log deviations from the steady state, reads:

\[
\bar{y}_t = (1 - g)\bar{c}_t + g\bar{g}_t
\]  
(B.11)

where \( g = \frac{\bar{T}}{\bar{y}} \) is the steady-state government spending-output ratio.

Government spending evolves according to the following AR(1) process:

\[
\bar{g}_t = \rho \bar{g}_{t-1} + \varepsilon_t.
\]  
(B.12)

**B.4. The Government Spending Multiplier of Normal Times**

The derivation of the government spending multiplier of normal times (i.e., when the nominal interest rate is free to adjust following government spending shocks) is shown in this subsection. To solve the model using the Method of Undetermined Coefficients, we make the following definitions for the response of variables to government spending shocks:

\[
\bar{y}_t = A_y \bar{g}_t
\]  
(B.13)

\[
\bar{c}_t = A_c \bar{g}_t
\]  
(B.14)

\[
\bar{\pi}_t = A_{\pi} \bar{g}_t
\]  
(B.15)

\[
\bar{R}_t = A_R \bar{g}_t
\]  
(B.16)

The government spending multiplier is then given by:

\[
\frac{dy_t}{dg_t} = A_y 
\]  
(B.17)

To see this, first start with \( \bar{y}_t = A_y \bar{g}_t \). Or, given the definition of log deviations \( \bar{w} = A_y \left( \frac{w - y}{y} \right) \). Re-arranging this equation gives: \( y_t = \left( \frac{A_y}{y} \right) g_t + \left( A_y - y \right) \bar{y} \). The multiplier is defined as \( \frac{dy_t}{dg_t} \). Therefore, taking first derivative with respect to \( g_t \) gives: \( \frac{dy_t}{dg_t} = \frac{A_y}{y} \bar{y} \). Finally, since \( g = \frac{\bar{T}}{\bar{y}} \), the spending multiplier will be given by \( \frac{dy_t}{dg_t} = \frac{A_y}{\bar{y}} \bar{y} \), which is equation (B.17).

We next show the derivation of the spending multiplier for each monetary policy rule.

**B.4.1. Inflation Targeting (IT)**

Under IT, we have the following system of equations:

\[
\bar{R}_t - E_t \bar{\pi}_{t+1} = \sigma (E_t \bar{c}_{t+1} - \bar{c}_t)
\]  
(B.18)

\[
\bar{\pi}_t = \beta E_t \bar{\pi}_{t+1} + \kappa [\nu \bar{y}_t + \sigma \bar{c}_t]
\]  
(B.19)

\[
\bar{y}_t = (1 - g)\bar{c}_t + g\bar{g}_t
\]  
(B.20)

\[
\bar{\pi}_t = 0
\]  
(B.21)
with (B.21) being the IT rule. These conditions can then be reduced to:

\[ A_R - \rho A_\pi = \sigma(\rho - 1)A_c \]  
(B.22)

\[ A_\pi = \beta \rho A_\pi + \kappa \left( \nu A_y + \sigma A_c \right) \]  
(B.23)

\[ A_y = (1 - g)A_c + g \]  
(B.24)

\[ A_\pi = 0 \]  
(B.25)

This is a system of 4 equations with 4 unknowns \((A_y, A_c, A_\pi, A_R)\). The solution of this system of equations gives:

\[ A_y^{IT} = \frac{\sigma}{\sigma + \nu(1 - g)} \]  
(B.26)

and the multiplier is \(A_y^{IT}/g\). We use this result to calculate the cumulative government spending multiplier, namely the effect of \(\hat{g}_t\) on \(\sum_{i=0}^{\infty} \beta^i \mathbb{E}_t \hat{y}_{t+i}\). To this end, notice that the partial effect of \(\hat{g}_t\) on \(\hat{y}_{t+i}\) is given by \(\frac{\sigma}{\sigma + \nu(1 - g)} \rho^i\). Then, the cumulative spending multiplier is given by:

\[ M^{IT} = \left( \frac{\sigma}{\sigma + \nu(1 - g)} \right) \frac{1}{1 - \beta \rho} \]  
(B.27)

which is condition (40) in the text.

**B.4.2. Taylor Rule**

When monetary policy is governed by a Taylor rule, we have:

\[ \widetilde{R}_t - \mathbb{E}_t \widetilde{\pi}_{t+1} = \sigma \left( E_t \hat{c}_{t+1} - \bar{c}_t \right) \]  
(B.28)

\[ \widetilde{\pi}_t = \beta \mathbb{E}_t \widetilde{\pi}_{t+1} + \kappa \left[ \nu \hat{\pi}_t + \sigma \hat{c}_t \right] \]  
(B.29)

\[ \hat{y}_t = (1 - g)\hat{c}_t + g \hat{\pi}_t \]  
(B.30)

\[ \widetilde{R}_t = \phi_\pi \widetilde{\pi}_t + \phi_y \hat{y}_t \]  
(B.31)

with (B.31) being the Taylor rule. As is standard in the literature, to insure determinacy, we assume \(\phi_\pi > 1\). For simplicity, we abstract from interest rate smoothing.

These conditions can then be reduced to:

\[ A_R - \rho A_\pi = \sigma(\rho - 1)A_c \]  
(B.32)

\[ A_\pi = \beta \rho A_\pi + \kappa \left[ \nu A_y + \sigma A_c \right] \]  
(B.33)

\[ A_y = (1 - g)A_c + g \]  
(B.34)

\[ A_R = \phi_\pi A_\pi + \phi_y A_y \]  
(B.35)

This is a system of 4 equations with 4 unknowns \((A_y, A_c, A_\pi, A_R)\). The solution of this system of equations gives:
\[ A_y^{TR} = \frac{\kappa \sigma (\phi_\pi - \rho) + \sigma (1 - \rho)(1 - \beta \rho)}{\kappa (\phi_\pi - \rho) \left[ \sigma + \nu (1 - g) \right] + [\sigma (1 - \rho) + \phi_y] (1 - \beta \rho)} \]  

(B.36)

and the corresponding cumulative spending multiplier is given by:

\[ M^{TR} = \left( \frac{\kappa \sigma (\phi_\pi - \rho) + \sigma (1 - \rho)(1 - \beta \rho)}{\kappa (\phi_\pi - \rho) \left[ \sigma + \nu (1 - g) \right] + [\sigma (1 - \rho) + \phi_y] (1 - \beta \rho)} \right) \frac{1}{1 - \beta \rho} \]  

(B.37)

which is condition (41) in the text.

B.4.3. Nominal GDP (NGDP) Targeting

To find the multiplier for NGDP targeting, we use the following system of equations:

\[ \overline{R}_t - \overline{E}_t \pi_{t+1} = \sigma (E_t \overline{c}_{t+1} - \overline{c}_t) \]  

(B.38)

\[ \pi_t = \beta \overline{E}_t \pi_{t+1} + \kappa [\nu \overline{\gamma}_t + \sigma \overline{c}_t] \]  

(B.39)

\[ \overline{\gamma}_t = (1 - g) \overline{c}_t + g \overline{\gamma}_t \]  

(B.40)

\[ \overline{\gamma}_t = \overline{\gamma}_{t-1} - \overline{\gamma}_t \]  

(B.41)

with (B.41) being the NGDP growth rate targeting rule. Notice also that, up to first-order approximation and under full adjustment, (B.41) corresponds also to the NGDP level targeting rule. Therefore, under these assumptions, the NGDPG and NGDPL targeting rules yield the same government spending multipliers.

Start by substituting (B.40) in (B.39) to obtain:

\[ \pi_t = \beta \overline{E}_t \pi_{t+1} + \frac{\kappa}{1 - g} \left[ \sigma + \nu (1 - g) \right] \overline{\gamma}_t - \sigma g \overline{\gamma}_t \]  

(B.42)

Then, substitute the NGDP targeting rule (B.41) into (B.42) to eliminate inflation and re-arrange:

\[ \left[ 1 + \beta + \frac{\kappa}{1 - g} [\sigma + \nu (1 - g)] \right] \overline{\gamma}_t = \overline{\gamma}_{t-1} + \beta \overline{E}_t \overline{\gamma}_{t+1} + \frac{\kappa \sigma g}{1 - g} \overline{\gamma}_t \]  

(B.43)

Unlike inflation targeting and a Taylor rule, with NGDP targeting, the system has one additional pre-determined variable (\( \overline{\gamma}_{t-1} \)). Therefore, \( \overline{\gamma}_t \) depends on both \( \overline{\gamma}_t \) and \( \overline{\gamma}_{t-1} \). To find the spending multiplier with NGDP targeting, we solve this second-order expectational difference equation.

We first define \( \Phi = 1 + \beta + \frac{\kappa}{1 - g} [\sigma + \nu (1 - g)] \) to obtain:

\[ \Phi \overline{\gamma}_t - \overline{\gamma}_{t-1} - \beta \overline{E}_t \overline{\gamma}_{t+1} - \frac{\kappa \sigma g}{1 - g} \overline{\gamma}_t = 0 \]  

(B.44)

Also, define \( \overline{x}_t = \overline{\gamma}_{t-1} \), which gives:
Then we obtain the following system of equations:

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\mathbb{E}_t y_{t+1} \\
\mathbb{E}_t x_{t+1}
\end{pmatrix}
= \begin{pmatrix}
\frac{\Phi}{\beta} & \frac{1}{\beta} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\mathbb{E}_t y_t \\
\mathbb{E}_t x_t
\end{pmatrix}
+ \begin{pmatrix}
\frac{-\kappa g}{\beta(1-g)} \\
0
\end{pmatrix} \bar{g}_t
\]

The eigenvalues of the first matrix on the right-hand side can be found by solving the following problem:

\[
\beta \lambda^2 - \Phi \lambda + 1 = 0 \quad (B.46)
\]

The solution gives:

\[
\lambda_{1,2} = \frac{\Phi \pm \sqrt{\Phi^2 - 4\beta}}{2\beta} \quad (B.47)
\]

The solution yields two positive eigenvalues; one eigenvalue that is larger than 1 (\(\lambda_1\)), and one eigenvalue that is less than 1 (\(\lambda_2\)). To prove that, assume first that \(\lambda_2 < 1\). Or:

\[
\Phi - \frac{\sqrt{\Phi^2 - 4\beta}}{2\beta} < 1 \quad (B.48)
\]

which, after re-arranging and squaring both sides, can be written as:

\[
(\Phi - 2\beta)^2 < \Phi^2 - 4\beta \quad (B.49)
\]

Simplifying gives:

\[
\Phi > 1 + \beta \quad (B.50)
\]

which, according to the definition of \(\Phi\) is true; therefore, \(\lambda_2 < 1\). Furthermore, the product of the two eigenvalues is \(\lambda_2 \cdot \lambda_1 = 1/\beta\), which is larger than 1. Since \(\lambda_2 < 1\), it must be the case that \(\lambda_1 > 1\).

Next, define:

\[
f_t = \bar{g}_t - \frac{1}{\beta \lambda_1} x_t \quad (B.51)
\]

which gives:

\[
f_t = \frac{\mathbb{E}_t f_{t+1}}{\beta} + \frac{\kappa g}{\beta(1-g)\lambda_1} \bar{g}_t \quad (B.52)
\]

Iterate forward on condition (B.52) to obtain:

\[
f_t = \frac{\kappa g}{\beta(1-g)\lambda_1} \sum_{i=0}^{\infty} \left( \frac{1}{\lambda_1} \right)^i \mathbb{E}_t \bar{g}_{t+i} \quad (B.53)
\]

Using the AR(1) process of government spending: \(\mathbb{E}_t \bar{g}_{t+i} = \rho^i \bar{g}_t\). Then, condition (B.53)
\[ f_t = \frac{\kappa \sigma g}{\beta(1-g)(\lambda_1 - \rho)} \hat{g}_t \] 

(B.54)

Condition (B.51) then gives:

\[ \hat{y}_t = \frac{1}{\beta \lambda_1} y_{t-1} + \frac{\kappa \sigma g}{\beta(1-g)(\lambda_1 - \rho)} \hat{g}_t \] 

(B.55)

According to equation (B.46), the product of the two roots is \(\lambda_2 \cdot \lambda_1 = 1/\beta\). Substituting this result in (B.55) to eliminate \(\lambda_1\) gives:

\[ \hat{y}_t = \lambda_2 \hat{y}_{t-1} + \frac{\kappa \sigma g}{(1-g)(1-\beta \rho \lambda_2)} \hat{g}_t \] 

(B.56)

To simplify the notation, let \(\lambda_2 = \lambda\). Then, we have:

\[ \hat{y}_t = \lambda \hat{y}_{t-1} + \frac{\kappa \sigma g}{(1-g)(1-\beta \rho \lambda)} \hat{g}_t \] 

(B.57)

Next, since \(\beta \lambda^2 + 1 = \Phi \lambda\), we can write:

\[ (1-\lambda)(1-\beta \lambda) = \lambda(\Phi - \beta - 1) \] 

(B.58)

But, using the definition of \(\Phi\), we obtain:

\[ \Phi - \beta - 1 = \frac{\kappa}{1-g} \left[ \sigma + \nu(1-g) \right] \] 

(B.59)

Substituting this result into condition (B.58) and re-arranging give:

\[ \lambda = \frac{(1-g)(1-\lambda)(1-\beta \lambda)}{\kappa[\sigma + \nu(1-g)]} \] 

(B.60)

Then, substitute (B.60) into (B.57) to obtain:

\[ \hat{y}_t = \lambda \hat{y}_{t-1} + \frac{(1-\lambda)(1-\beta \lambda)}{(1-\beta \rho \lambda)} \frac{\sigma g}{\sigma + \nu(1-g)} \hat{g}_t \] 

(B.61)

In what follows, we use condition (B.61) to derive the cumulative government spending multiplier. By successive iterations on (B.61), for any period \(t+i\), we have:

\[ \hat{y}_{t+i} = \lambda^{i-1} \hat{y}_{t-1} + \frac{\sigma g}{\sigma + \nu(1-g)} \frac{(1-\lambda)(1-\beta \lambda)}{(1-\beta \rho \lambda)} \left[ \lambda^i + \rho \lambda^{i-1} + \rho^2 \lambda^{i-2} + \ldots + \rho^i \right] \hat{g}_t \] 

(B.62)

which, after factoring out \(\rho^i\), can also be re-written as:
\[
\hat{y}_{t+1} = \lambda^{i+1}\hat{y}_{t-1} + \frac{\sigma g}{\sigma + \nu(1-g)} \frac{(1-\lambda)(1-\beta\lambda)}{(1-\beta\rho\lambda)} \rho^i \left[ (\lambda/\rho)^i + (\lambda/\rho)^{i-1} + (\lambda/\rho)^{i-2} + \ldots + 1 \right] \hat{g}_t \tag{B.63}
\]

or, using the sum of a geometric series:
\[
\hat{y}_{t+1} = \lambda^{i+1}\hat{y}_{t-1} + \frac{\sigma g}{\sigma + \nu(1-g)} \frac{(1-\lambda)(1-\beta\lambda)}{(1-\beta\rho\lambda)} \rho^i \frac{1-(\lambda/\rho)^i}{1-(\lambda/\rho)} \hat{g}_t \tag{B.64}
\]

The partial effect of \(\hat{g}_t\) on \(\hat{y}_{t+i}\) is given by:
\[
\frac{\partial \hat{y}_{t+i}}{\partial \hat{g}_t} = \frac{\sigma g}{\sigma + \nu(1-g)} \frac{(1-\lambda)(1-\beta\lambda)}{(1-\beta\rho\lambda)} \rho^i \frac{1-(\lambda/\rho)^i}{1-(\lambda/\rho)} \tag{B.65}
\]

Then, the cumulative spending multiplier can be written as:
\[
M_{NGDPG} = M_{NGDPL} = \left( \frac{\sigma}{\sigma + \nu(1-g)} \right) \frac{(1-\lambda)}{(1-\beta\rho)(1-\beta\rho\lambda)} \tag{B.66}
\]

which is condition (42) in the text.

**B.5. The Government Spending Multiplier of Liquidity Trap**

When the nominal interest rate is fixed, as may happen at the zero-lower bound (ZLB), \(A_R = 0\) by definition. Therefore, letting \(p\) be the probability that the nominal interest rate does not change next period, we have the following system of 3 equations and 3 unknowns \((A_y, A_c, A_\pi)\):
\[
pA_\pi = \sigma(1-p)A_c \tag{B.67}
\]
\[
A_\pi = \beta p A_\pi + \kappa [\nu A_y + \sigma A_c] \tag{B.68}
\]
\[
A_y = (1-g)A_c + g \tag{B.69}
\]

Conditions (B.67)-(B.69) pin down the value of the government spending multiplier of a constant nominal interest rate. These three equations correspond to the Euler equation, Phillips curve and the resource constraint, and they are the same for all monetary policy rules. Therefore, the value of the spending multiplier can be derived without using a specific monetary policy rule or the adjustment variables. For this reason, the government spending multiplier is the same for all monetary policy rules.

Finally, as in Christiano et al. (2011) and Carlstrom et al. (2014), among others, the determinacy condition requires the denominator of the government spending multiplier to be positive (namely, \(\sigma[(1-p)(1-\beta p) - \kappa p] - \kappa p \nu (1-g) > 0\)), and we make the analysis under this assumption. It should be noted, though, that the result that all monetary policy rules deliver the same government spending multiplier in liquidity traps does not require this assumption.